Volume XVII. *July, 1903.* *Number 1.*

THE

PHYSICAL REVIEW.

ULTRA-VIOLET ROTARY DISPERSION.

By P. G. Nutting.

THE power of rotating the plane of polarization of transmitted radiation is particularly interesting in the region of the very short wave-lengths. In the first place it affords an indication of what the limiting value of the rotation would be for infinitely short waves. From theoretical considerations we might expect this limiting value to be either zero or infinity, possibly a finite constant. Again, rotary dispersion in the ultra-violet affords valuable data for the theory of rotation in the neighborhood of an absorption band. Theoretically, we should have a greatly enhanced rotation near every absorption band and a large rotation of opposite sense in the middle of the band. Now all transmission spectra end sooner or later in the ultra-violet in an absorption band, so that the ultra-violet region is an excellent one for obtaining the data desired.

Previous work in the ultra-violet has been confined to quartz and sodium chlorate. Soret and Sarasin[1] have observed the rotation of quartz out as far as the cadmium line 26, wave-length 214.3 $\mu\mu$, where they found the rotation per millimeter thickness 236 degrees, while in the yellow it is only about 21 degrees and in the infra-red about ten. Guye[2] observed the rotation of sodium chlorate out as far as wave-length 250 $\mu\mu$. The rotation of this substance is

[1] Compt. rend., 95, 636, 1882.
[2] Compt. rend., 108, 348, 1889.

small, increasing from three degrees in the yellow to fifteen degrees at 250.

Rotation in the neighborhood of an absorption band has been observed by Schmauss[1] working with the magnetic rotation of cyanine and other dyestuffs in alcohol solution, and by Cotton[2] who observed the structural rotation of copper and chromium tartrates. Zeeman[3] has observed the magnetic rotation of sodium vapor up to and within the lines. Several other observers have worked on anomalous rotation, that is, rotation which decreases with decreasing wave-length.

The methods available for work in the ultra-violet and in regions of strong absorption are but limited in number. Previous work has been done by visual methods with a fluorescent eyepiece. Guye used a quartz-wedge polariscope with his eyepiece. But visual methods fail when the transmitted radiation is excessively feeble and one must resort to the cumulative methods of photography. Glan or Foucault nicols can be used out as far as 230 $\mu\mu$, where the calcite begins to absorb. Schmauss inserted a block of quartz and a plate of gypsum with his rotating column of fluid. This method might be used photographically but gypsum is opaque in the ultra-violet and suitable transparent biaxial crystals are not available. A much simpler and entirely satisfactory method was suggested by the apparatus used by Minor[4] for determining the refractive and absorptive indices of the metals in the ultra-violet. This method consists in inserting with the rotating column a pair of quartz rotation wedges. Such wedges produce interference bands which are easily photographed. Inserting a rotating body with the wedges produces a displacement of these bands and the amount of the shift is an accurate measure of the amount of rotation introduced.

APPARATUS AND METHOD.

It is, of course, a great saving of time in exposing and measuring to be able to photograph the rotation of the whole spectrum on a single plate. This was found to be feasible experimentally. To

[1] Ann. der Physik, 2, 280, 1900.
[2] Jour. de Ph. (3), 5, 237, 1896.
[3] V. K. Ak. van Wet., 1902–3, pp. 6–11.
[4] Ann. der Physik (4), 10, 581, 1903.

accomplish this we must get the interference bands and spectrum lines in focus at the same place, that is the rotating wedges must be placed at the slit or an image of the slit of the spectroscope used in producing the spectrum. The final arrangement adopted is shown in the figure.

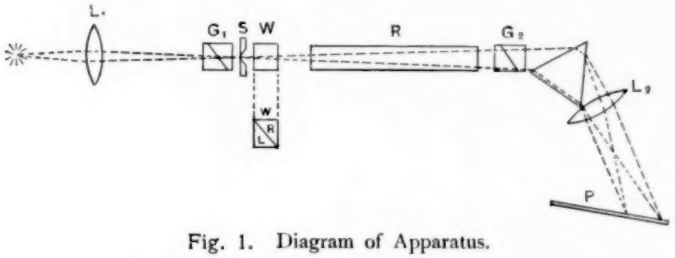

Fig. 1. Diagram of Apparatus.

An image of the spark source is thrown on the slit S and a spectrum formed on the photographic plate P. Before entering the slit, the light is polarized by the Glan nicol G_1 and immediately afterward passes through the rotation wedges W. A second Glan nicol G_2 is placed near the prism and the tube of rotating solution inserted between this nicol and the wedge pair. The distance $L_1 - S$ and $S - P$ are made very great so that the light is very nearly parallel as it passes through the wedges and rotating substance. The nicols were used parallel so that the central straight band was dark on the photographic plate.

A spark between ectrodes made of an alloy of zinc and aluminium was used as a source of radiation through all the work. The spark spectrum of this alloy contains a convenient number of lines very evenly distributed. A crystal of some sodium salt was held in the spark from time to time. This not only brought up the photographic intensity in the yellow but so increased the possible length of the spark that much more capacity could be used and the intensity of the source increased many times. The Glan nicols were 20 mm. cubes and were found to be admirably adapted to this use. The nicols do not require very accurate adjustment for this kind of work. The slit was set permanently at about 0.3 mm. opening and firmly mounted on a massive metal plate on which the wedges could be mounted and very accurately adjusted. The slit was fitted with a transverse wire cross hair that could be raised and

lowered until its image very nearly bisected the central dark interference band on the photographic plate. It is then opposite that part of the wedges where they are of equal thickness. The wedges were cut to order, one of right and one of left rotating quartz 16 mm. thick and 20 mm. square. The tube *R* for holding the rotating liquid was closed at the ends with a pair of very carefully matched quartz plates, right and left rotating, giving no appreciable resultant rotation. These plates were 25 mm. in diameter and 2.872 mm. thick. A number of different tubes were used, some 5 cm. and some 60 cm. in length, according to the capacity and rotating power of the liquid being investigated. The tube most in use was one of glass 37 cm. in length with the ends very carefully ground to parallelism and the quartz end plates cemented on. The tube and wedge pair require very careful adjustment, otherwise the interference bands would not be regular and sharply defined. These adjustments were made by means of the reflected image of the slit, using an acetylene source. All the quartz parts of the apparatus as well as the Glan nicols, were specially ordered from Messrs. Steeg and Reuter and were of most admirable workmanship. The most difficult and important adjustment of all is finding the proper position and angle for the photographic plate in order that the image of the cross wire and the interference bands may be sharp and straight throughout the whole spectrum. The proper angle having been determined, the plate holder was provided with permanent ways and a micrometer screw. The prism was set at minimum deviation for the end of the visible spectrum, wave-length 400 $\mu\mu$. Zero plates were taken with the tube filled with water or alcohol, with the tube empty and with the tube entirely removed. After the rotating substance had been inserted, a second photograph was taken. This showed the interference bands displaced up or down along the spectrum lines and the amount and direction of this displacement shows the amount and direction of the rotation at each wave-length. The displacements were measured with a Gaertner micrometer microscope refitted with a low power microscope and glass bed plate. The very highest grade of photographic plates, made of very plane glass and with films free from spots are of course essential for this work. Cramer's instantaneous isochro-

matic plates were found to be entirely satisfactory. Metol-hydrochinone developer gives a hard, clear negative excellent for measuring. Over-exposure and over-development made the bands still sharper and narrower. The time of exposure varied from two minutes to half an hour.

Observed Data on Rotary Dispersion.

Out of the large number of rotating substances available, some thirty were tried as promising transparency in the ultra-violet and of these, eight were found to be suitable for observation. The results are given in the order in which they were obtained. Of the other promising substances tried, quinine (sulphate) in water solution was found to become densely opaque just beyond the visible violet. The absorbed ultra-violet quickly precipitated the salt from a saturated solution. Morphine sulphate showed similar properties. Santonin, which possesses the extraordinary rotation of 200 degrees in the yellow and 535 degrees at the end of the visible violet,[1] becomes strongly absorbent just beyond. The various iron double tartrates, which absorb nearly the entire spectrum, show but moderate rotating power in their transmission bands.

Cane Sugar, $C_{12}H_{22}O_{11}$.—This, in concentrated solution, begins to absorb just beyond the visible, but by taking a very dilute solution and making a long exposure, the rotation was obtained out as far as wave-length 252 $\mu\mu$. The concentration used was 3.45 grams of sugar to 100 grams of solution, density 1.013, tube length 77.32 cm., temperature 18 degrees. In the table of data below, the column marked λ refers to wave-lengths, that marked

λ	180° mm.	Ref. mm.	Ref. deg.	Null.	Ob. Rotn.	Sp. Rotn.
589	3.402	1.920	101.6	83.6	18.0	66.8
467	2.134	1.311	110.7	79.6	31.1	115.2
425	1.730	1.094	113.8	77.2	36.6	134.1
395	1.492	0.954	115.0	75.4	40.6	150.4
358	1.182	0.799	121.7	72.6	49.1	182.0
334	0.992	0.708	128.7	70.8	57.9	214.8
309	0.810	0.642	142.6	68.5	74.1	274.8
281	0.637	0.588	165.1	65.6	99.5	368.9
256	0.527	0.574	196.4	62.6	133.8	496.0
252	0.486	0.554	205.7	61.9	143.8	533.5

[1] Nasini, Ac. dei Lincei, 3, 13, 1882.

180° gives the width of an interference band in mm. That marked Ref. (mm). gives the distance from the image of the cross wire to the center of the interference band use for reference. Under Ref. (deg.) is given the same distance expressed in degrees. "Null" is the corresponding distance before rotation occurred. Then follow observed rotation and specific rotation.

From wave-length 358 on, the second band was used for reference instead of the first. The distances given in the third column were obtained by subtracting the width of a band from the observed distances. The specific rotation is calculated from the observed rotation by means of the formula

$$R = r/lpd,$$

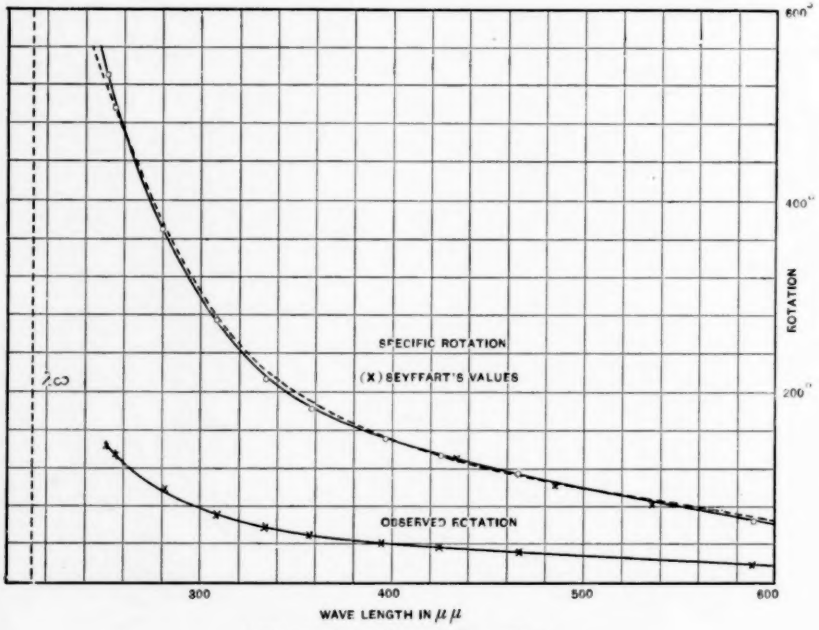

Fig. 2. Rotation of Cane Sugar.

where l is the length of the observed column of solution expressed in decimeters, d is the density of the solution and p is the ratio of the weight of the dissolved solid to the weight of the solution in a given volume. It is the concentration of the solution divided by its density. The factor l/pd merely corrects for the presence of the neutral solvent.

The dispersion curve is remarkably uniform and shows steadily increasing values toward the shorter wave-lengths, rising well out in the absorption band at 250 to a rotation of 560°. No exterior limit to the absorption band could be observed; it probably continues through the remainder of the spectrum. Some previous observations of Seyffart in the visible spectrum are given.

Camphor, $C_{10}H_{16}O$.—Camphor has a strong absorption beginning just beyond the visible. Readings were obtained as far as 334. The material used appeared very pure, but as camphor is such an

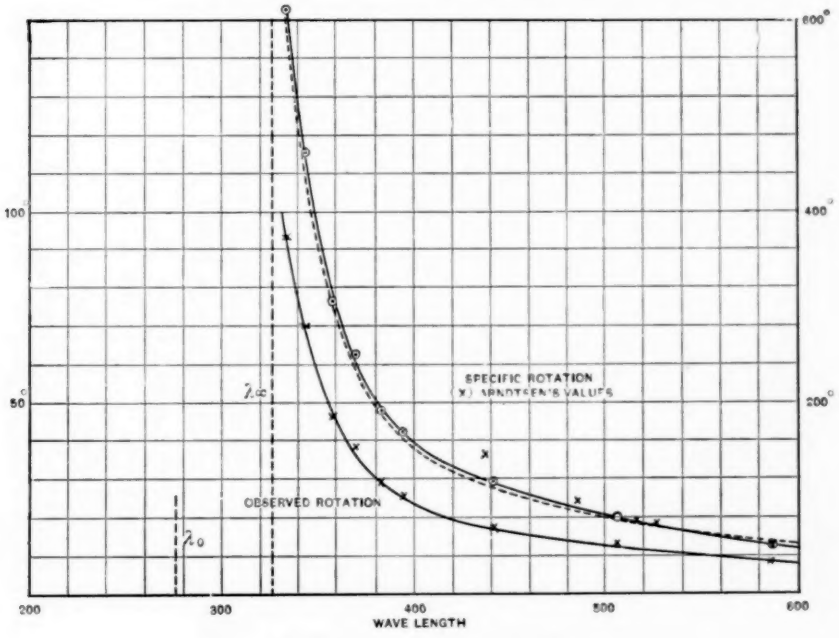

Fig. 3. Rotation of Camphor.

excellent solvent for organic impurities, it was twice distilled before dissolving in freshly distilled ethyl alcohol for use. The concentration used was 34.70 grams of camphor to 100 grams of solutional density 0.864, temperature 18°.

The curve is an excellent example of the dispersion in the neighborhood of a broad band. The dispersion remains regular and normal up to near where the absorption begins and there increases very sharply. The essentially different value obtained by Arndtsen

at wave-length 438 might easily have been caused by a slight shifting of the absorption band due to a trace of impurity or isomeric camphor. Camphoric acid was tried but it transmitted only the visible spectrum and its rotation showed no unusual features.

λ	180° mm.	Ref. mm.	Ref. deg.	Null.	Ob. Rotn.	Sp. Rotn.
589	2.505	1.277	91.62	83.61	8.01	52.4
506	1.880	0.985	94.4	81.1	13.3	87.2
441	1.425	0.760	96.0	78.1	17.9	117.2
395	1.095	0.615	101.1	75.4	25.7	168.1
383	1.030	0.594	103.9	74.6	29.3	191.9
370	0.940	0.515	111.9	73.5	38.4	251.4
358	0.897	0.593	118.9	72.6	46.3	303.0
344	0.786	0.620	142.0	71.6	70.4	462.0
334	0.706	0.644	164.3	70.8	93.5	612.5

Limonene and *Pinene*.—Two of the principal components of turpentine, dextro-limonene and lævo-pinene, were tested and found to absorb slightly at wave-length 420 and quite completely even in columns but 4 cm. thick at 390. The observed rotations are here recorded, however, on account of the striking dissimilarity in the

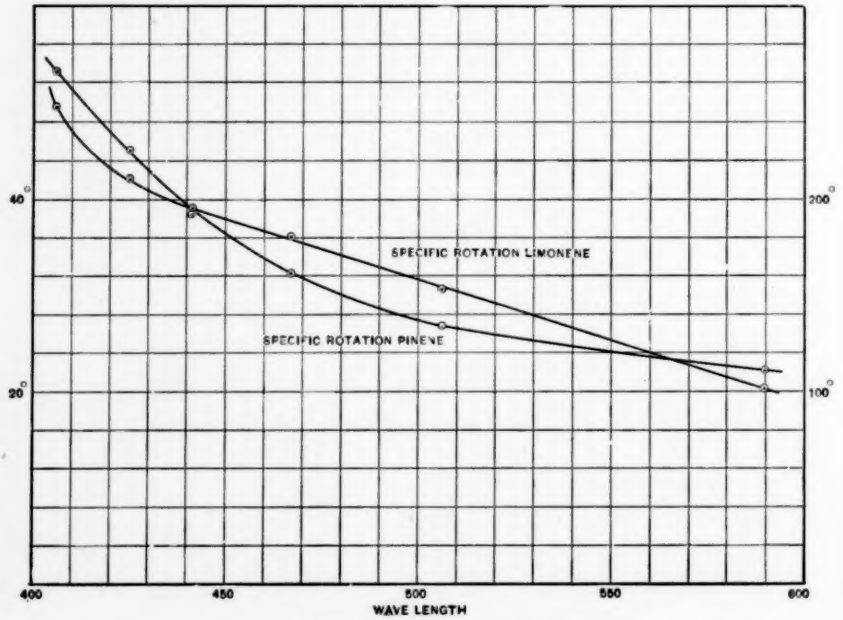

Fig. 4. Rotation of Limonene and Pinene.

dispersion curves of the two substances and the departure of both curves from the normal form that we should expect.

Specific Rotation.

λ	Limonene.	Pinene.
589	100.7	22.35
506	153.6	26.92
467	180.1	32.20
441	194.5	37.64
425	211.4	45.01
406	248.0	53.35

Neither curve is of the normal type as are those of sugar and camphor. The curvature of the pinene curve is remarkably uniform while that of the limonene is very small throughout most of its length.

Lactose, Milk Sugar, $C_{12}H_{22}O_{11}+H_2O$.—The lactose used was the chemical preparation supplied by Merck and proved quite trans-

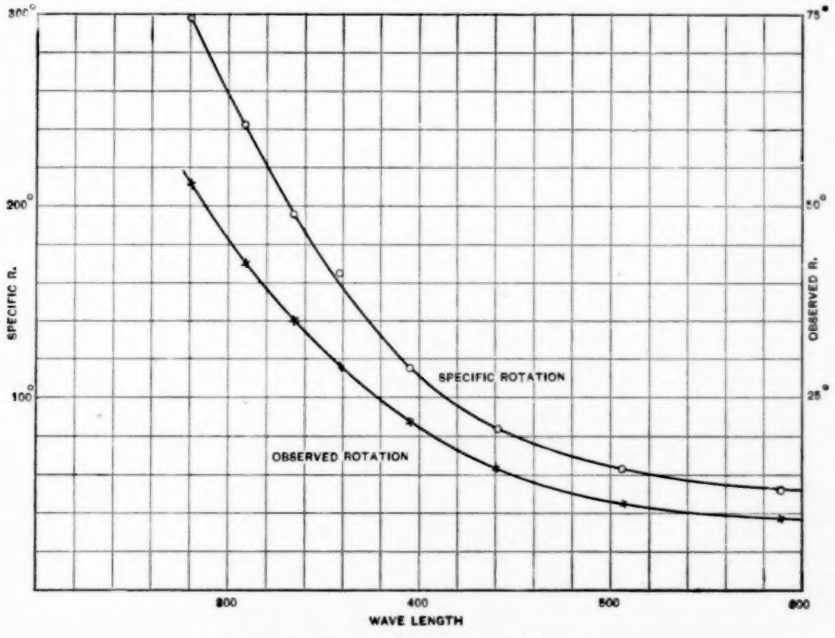

Fig. 5. Rotary Dispersion of Lactose.

parent in the ultra-violet. Readings were obtained as far as 281 $\mu\mu$, absorption being noticeable beyond 350. A water solution was

used having the density 1.025 and containing 15.81 per cent. of lactose. The temperature was 19°.

λ	180° mm.	Ref. mm.	Ref. deg.	Null.	Ob. Rotn.	Sp. Rotn.
589	2.987	1.398	84.15	95.4	9.25	52.4
506	2.250	1.047	83.83	95.0	11.17	63.3
441	1.695	0.744	77.0	94.9	15.9	90.1
395	1.219	0.444	73.1	94.9	21.8	114.2
358	1.020	0.377	66.6	95.8	29.2	165.3
334	0.855	0.294	61.9	96.9	35.0	196.4
309	0.707	0.224	57.6	100.4	42.8	242.4
281	0.581	0.171	55.2	108.1	52.9	299.5

The rotary dispersion is small and uniform, such as would correspond with a broad absorption band with diffuse edges. The curves are normal and typical.

Maltose and *Glucose*, $C_6H_{12}O_6$.—The purest chemically prepared glucose obtainable was slightly yellow in solution, absorption began

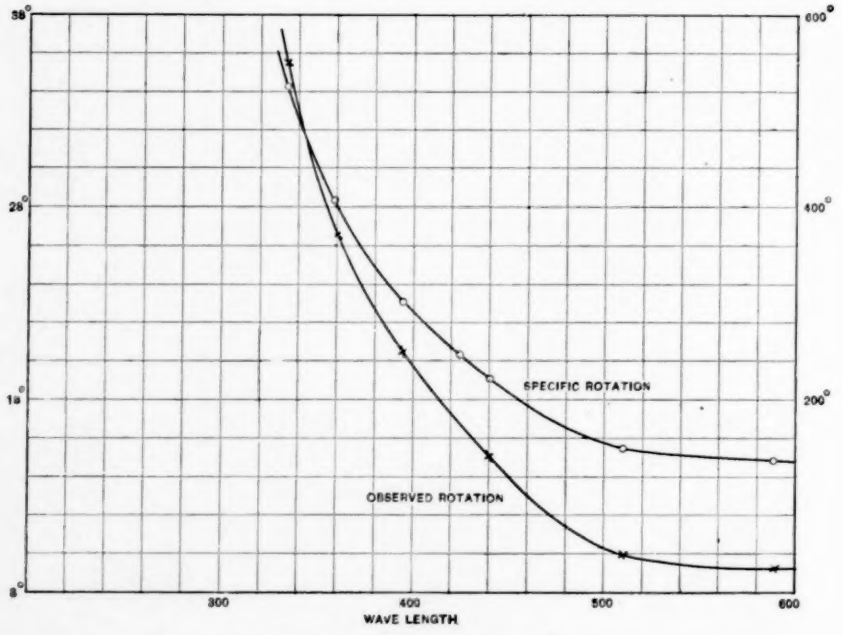

Fig. 6. Rotary Dispersion of Maltose.

in the green and became very strong in the violet. Shaking up the solution with bone-black and filtering through sand and charcoal

did not improve its transparency. Maltose proved more transparent and with a very dilute solution readings were obtained out to 334 $\mu\mu$. The density of the solution was 1.005 and it contained 1.794 per cent. of maltose. Tube length 37.59 cm., temperature 19°.

λ	180° mm.	Ref. mm.	Ref. deg.	Null.	Ob. Rotn.	Sp. Rotn.
589	2.918	1.394	86.10	95.41	9.31	137.5
506	2.200	1.038	84.9	95.0	10.1	149.3
441	1.684	0.750	79.9	94.9	15.0	221.9
425	1.526	0.662	78.2	94.9	16.7	246.8
395	1.290	0.537	74.4	94.9	20.5	303.0
358	1.023	0.389	68.4	95.8	27.4	405.1
334	0.85	0.29	61.4	96.9	35.5	524.8

The dispersion curves are of the ordinary type except that there is a slight irregularity in the green and just beyond the visible such as would be caused by a shoulder in the absorption curve in the blue.

Tartaric Acid, $H_2C_4H_4O_6$.—This, as well as nearly all of its salts, is quite transparent in the ultra-violet when very pure. Measurements were made out as far as wave-length 281 without difficulty. A water solution was used having a density 1.236 and containing 28.625 per cent. of the acid crystals.

λ	180° mm.	Ref. mm.	Ref. deg.	Null.	Ob. Rotn.	Sp. Rotn.
589	2.982	1.362	82.35	95.4	13.05	9.82
506	2.283	1.068	84.3	95.0	10.7	8.06
467	1.894	0.895	85.0	94.9	9.9	7.49
425	1.555	0.747	86.4	94.9	8.5	6.39
395	1.357	0.661	87.7	94.9	+ 7.2	+ 5.42
358	1.055	0.644	109.8	95.8	−14.0	−10.58
334	0.864	0.694	144.6	96.9	47.7	35.82
309	0.721	0.829	269.0	100.4	168.6	126.8
281	0.629	1.522	436.7	108.1	328.6	247.4

The rotary dispersion of tartaric acid is remarkable for the reversal of the sense of the rotation and for the abrupt change from a small constant positive rotation in the visible spectrum to a very large negative rotation in the ultra-violet. The reversal of the sense of the rotation at a certain wave-length has long been known and studied. In less concentrated solutions the reversal occurs in the visible spectrum. Hence at a given wave-length, within certain

limits, the sense of the rotation reverses at a certain concentration. To account for these exceptional properties of tartaric acid it is usual to assume that the molecules are not all alike in structure. But for the physical discussion of the rotary dispersion we must regard a solution of this substance as of fixed definite composition which is of course independent of the wave-length.

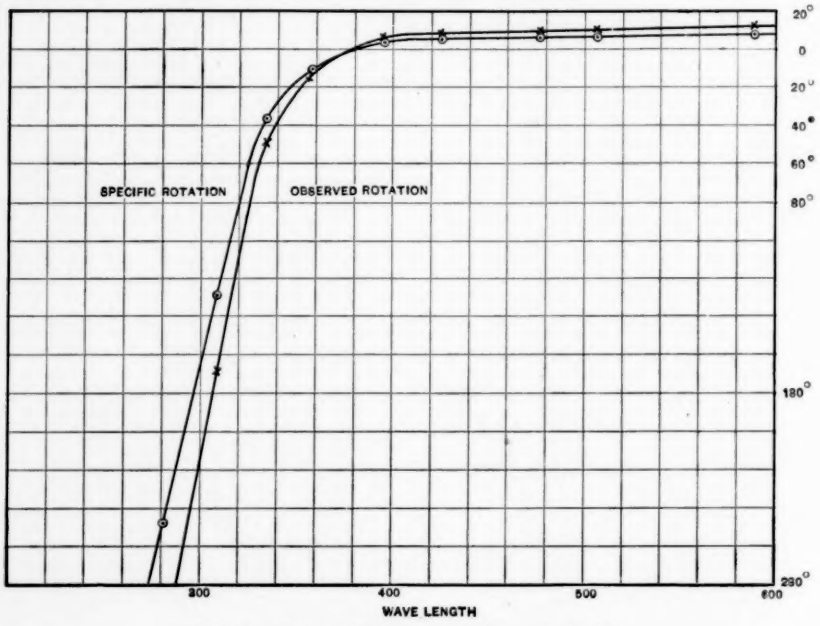

Fig 7. Rotary Dispersion of Tartaric Acid.

Ammonium Tartrate $(NH_4)_2C_4H_4O_6$. — A 16.24 per cent. water solution was used. Density, 1.070; temperature, 19°; tube length, 37.59 cm.

λ	180° mm.	Ref. mm.	Ref. deg.	Null.	Ob. Rotn.	Sp. Rotn.
589	3.123	1.269	73.2	95.4	22.2	34.2
506	2.351	0.858	65.7	95.0	29.3	44.9
467	1.977	0.649	59.1	94.9	35.8	54.9
425	1.609	0.459	51.4	94.9	43.5	66.7
395	1.356	0.328	43.6	94.9	51.3	78.6
358	1.067	0.201	34.0	95.8	61.8	94.7
334	0.921	0.142	28.8	96.9	68.1	104.3
309	0.749	0.102	24.5	101.2	76.7	117.4
281	0.578	0.061	18.4	105.5	86.6	132.7

The dispersion curves for ammonium tartrate are quite regular and are rather an extreme type of very small rate of change of dispersion like the turpentine components. Camphor and tartaric acid are the opposite extreme.

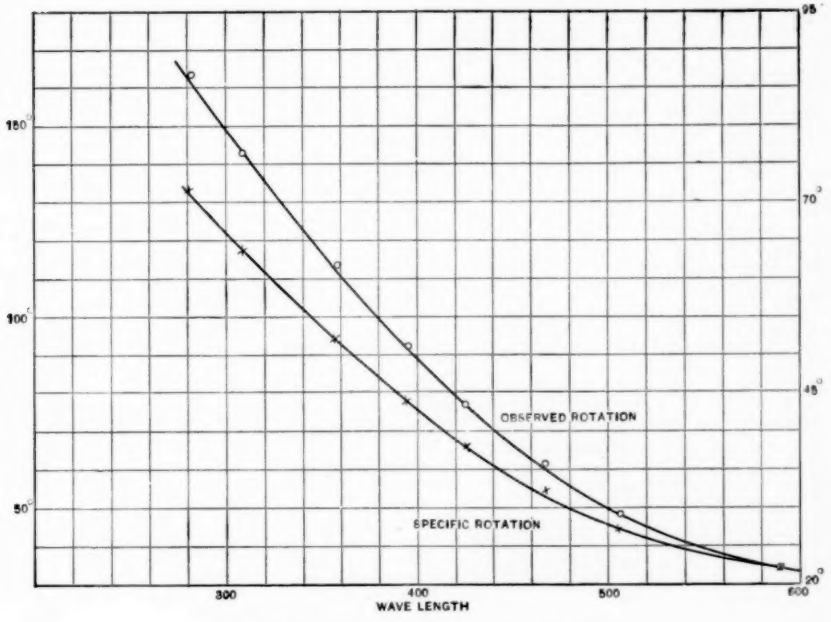

Fig. 8. Dispersion of Ammonium Tartrate.

Potassium Sodium Tartrate, Rochelle Salt, $KNaC_4H_4O_6 + 4H_2O$. — Water solution, density, 1.078, containing 18.75 grams of salt to 100 grams of solution. Absorption band intense beyond 300 $\mu\mu$ and extends down into the visible blue.

λ	180° mm.	Ref. mm.	Ref. deg.	Null.	Ob. Rotn.	Sp. Rotn.
589	3.007	1.320	79.0	95.4	16.4	29.7
506	2.243	0.956	76.8	95.0	18.2	33.0
467	1.898	0.744	70.5	94.9	20.4	36.9
425	1.553	0.593	68.7	94.9	25.6	46.4
395	1.345	0.494	66.1	94.9	28.8	32.2
358	1.065	0.366	61.8	95.8	34.0	62.3
334	0.851	0.274	58.8	96.9	38.1	69.0
309	0.68	0.22	58.4	100.4	42.0	76.1

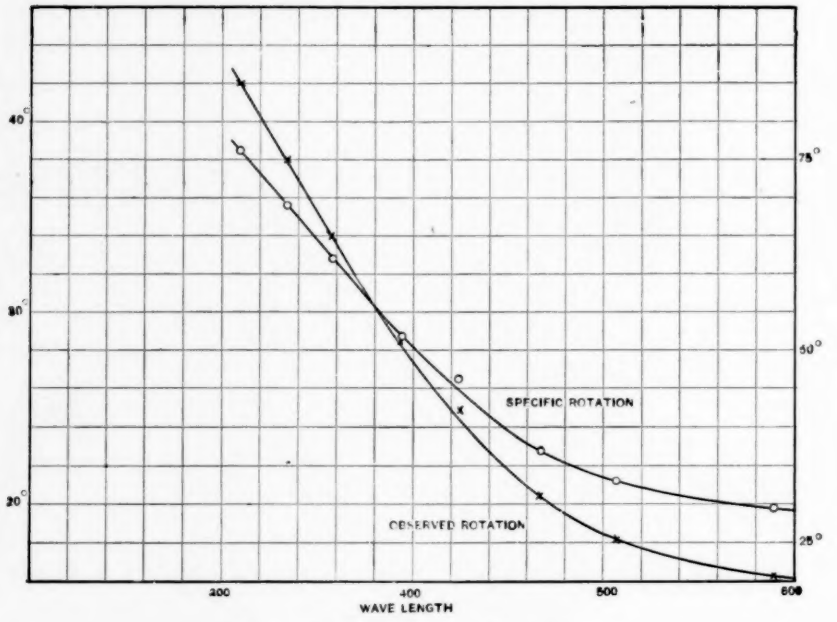

Fig. 9. Potassium Sodium Tartrate.

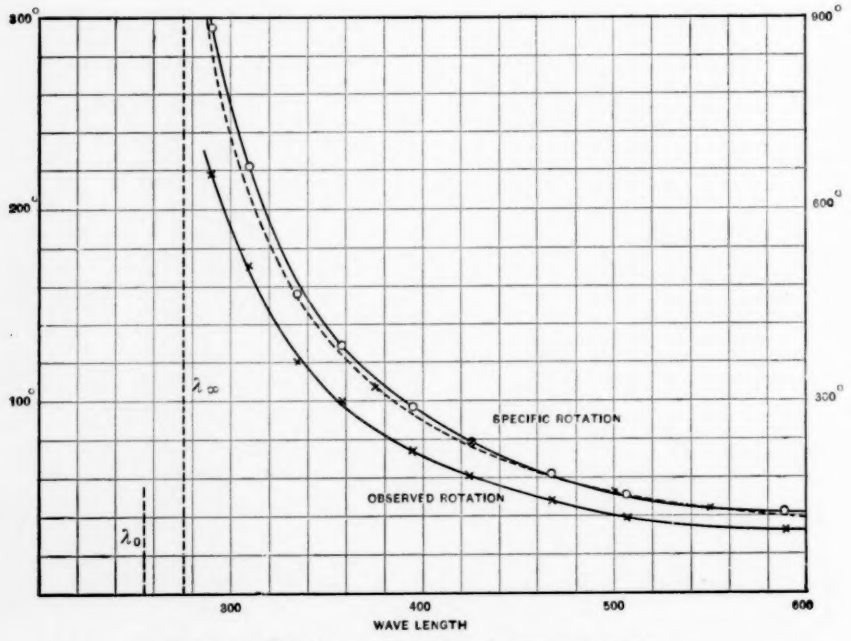

Fig. 10. Rotation of Potassium Antimony Tartrate.

The dispersion curves are of the regular form, the dispersion being unusually constant in the ultra-violet, indicating perhaps that the maximum of the absorption is still far distant or else that the absorption does not become very intense.

Potassium Antimony Tartrate, Tartar Emetic, $2K(SbO)C_4H_4O_6 + H_2O$. — Water solution, density 1.032, containing 6.635 per cent. of salt. Absorption perceptible only in the ultra-violet.

λ	180° mm.	Ref. mm.	Ref. deg.	Null	Ob. Rotn	Sp. Rotn.
589	2.977	1.044	63.3	95.4	32.1	124.8
506	2.247	0.681	54.6	95.0	40.4	157.0
467	1.867	0.480	46.4	94.9	48.5	188.4
425	1.527	0.283	33.4	94.9	61.5	238.8
396	1.306	0.146	29.14	94.9	74.7	290.0
358	1.039	1.013	175.7	95.8	100.1	388.8
334	0.868	0.751	155.7	96.9	121.2	471.0
309	0.722	0.441	110.4	101.2	170.8	663.5
290	0.644	0.24	167.3	105.5	228.2	886.5

The specific rotatory power is large even in the visible, but in the edge of the absorption band reaches the remarkable value of nine hundred degrees. The rate of increase of dispersion is remarkably uniform, like that of pinene and in marked contrast to that of camphor and tartaric acid.

These ten dispersion curves show very clearly that they could not be made congruent by merely altering the scale on which they are plotted, nor could they be made even approximately straight lines by plotting logarithms of wave-lengths instead of wave-lengths. The cane sugar, lactose and ammonium tartrate curves have somewhat the form of the exponential, but camphor, limonene, maltose and potassium antimony tartrate curves could not be represented by an exponential or logarithmic function even approximately. All the curves are of the form of parts of hyperbolas of higher degree

$$R f(\lambda) = \text{const. or } R f(\lambda) = \varphi(\lambda).$$

But if we are to use an algebraic dispersion formula, we see from the irregularities of the curves that it must contain at least four constants. For instance, consider the simplest possible form, the Laurent expansion of the actual function whatever that is, namely:

$$R = A + B/\lambda + C/\lambda^2 + \cdots.$$

A curve like Fig. 10 could be very approximately represented by the first three terms of this series since its curvature is quite uniform. Dispersion like that of cane sugar (Fig. 2) could be represented fairly well by four terms of the series since its change of curvature is uniform. But to represent the dispersion of camphor (Fig. 3) or of tartaric acid (Fig. 7) would require at least five terms, since the curvature is at least quadratic in λ. Evidently no mere expansion formula will satisfactorily represent such dispersion as we have here to deal with. For use in determining the constants of dispersion formulas the following values of the rotation were taken from the plotted curves:

λ	Sugar.	Camph.	Limon.	Pinene.	Lact.	Malt.	Tart. Acid.	Am. Tart.	KNa Tart.	KSb Tart.
600	62.1	46.1	95.6	21.8	51.9	135.9	9.4	32.9	29.4	125
575	71.1	54.1	111.0	22.9	53.6	138.3	8.9	35.4	29.8	128
550	80.3	62.0	126.8	24.2	56.0	141.9	8.4	38.2	30.7	135
525	89.7	71.4	142.2	25.6	59.5	145.8	7.9	41.7	31.8	143
500	99.8	81.7	158.0	27.6	64.6	157.0	7.5	45.8	33.6	158
475	110.3	94.3	173.9	30.8	71.2	176.5	7.0	51.1	35.9	177
450	122.2	109.8	189.6	36.4	80.1	207.7	6.6	57.8	39.6	204
425	134.6	129.7	211.0	44.9	92.2	246.8	6.1	65.9	44.7	238
400	149.9	158.6	290	56.1	110.1	291.5	6.0	75.4	50.6	281
375	167.4	221.7			137.3	350.7	1.4	86.0	57.3	334
350	192.9	378.3			171.4	440.6	16.8	97.5	67.9	418
325	252.2				210.9	586	56.8	109.5	78.6	548
300	297.7				258.3		166.0	121.4	88.9	758
275	394.4				310		296.8	142		1020
250	543.0									

THE MECHANISM OF ROTATION.

In attempting to develop a consistent possible mechanism of rotation and rotary dispersion, we are led at the outset to distinguish three entirely different rotatory processes, independent of one another and superposable upon each other without interference. These three processes are concerned with the magnetic rotation exhibited by all substances, the structural rotation of some crystals and torsionally strained solids and the molecular rotation shown by certain organic fluids and solids. Naturally active fluids and crys-

tals in a magnetic field rotate just as much on their own account as though the field were not there. And in the very rare instances in which active fluids crystallize so as to rotate structurally as well, the rotation due to each mechanism may be shown to exist independently of the other. It is significant that empty ether is not even magnetically rotating, the presence of matter is necessary to give the magnetic field an effective hold on the light waves.

Dissolve in a transparent solvent a substance having a narrow absorption band and the magnetic rotation of the solution will be greatly enchanced in the neighborhood of the absorption band. But if the solvent be itself rotationally active and the absorbing solute non-active, the rotation of the solution will not differ from that of the pure solvent. The absorbing particles must form a part of the rotating molecule itself before they can affect the rotation. In crystalline rotation it is sufficient that the added particles form an integral part of the crystalline structure.

Molecular rotation is exhibited by such crystals as can occur in twin pairs, one of these forms rotating right-handedly and the other an equal amount left-handedly. The forms of these twin pairs are images of each other, but only such images as are not superposable on each other. In this case the rotation appears to be due entirely to the manner in which the molecules are packed together, for structural rotation is always lost on fusion or solution. It may be imitated by a pile of mica plates arranged with the principal section of each plate turned 60°, 90° or 120° from the last. But the mechanism of the rotation caused by the mica plate pile can hardly be the same as that of active crystals.

The molecules of active fluids exhibit the same asymmetry of structure as to active crystals. The four atoms or radicles at the extremities of the four carbon bonds are such that they are capable of forming two essentially different molecular structures. These twin pairs are non-superposable images of each other. An active fluid or solution has of course the same rotating power in every direction just as a pile of active crystals would, and probably for about the same reason. Change of state does not affect molecular or magnetic rotation; even in vapor form the specific rotation is the same.

In each of the three kinds of rotation it has been shown that a light wave is resolved into two circularly polarized components in traversing the rotating medium. Fresnel in 1823 showed the existence of circular double refraction in quartz, von Fleischl [1] tested active fluids and recently Brace [2] has shown the same to exist in magnetically active glass. Now it may be shown mathematically that the resultant of two circularly polarized waves of equal amplitude, but different (not complex) velocities is a plane polarized wave whose plane of polarization rotates as the wave advances. Hence any substance that has the power of breaking up a linear vibration into its circular components will rotate the plane of vibration, since the one essential for having this power is that right and left circularly polarized waves are transmitted with unequal velocities. If the velocities are real, say V_1 and V_2, then the rotation per unit length, expressed in radians is

$$\frac{\pi}{T}\left(\frac{1}{V_2}-\frac{1}{V_1}\right) \text{ or } \pi\left(\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right)$$

where T is wave-period and λ wave-length. In terms of circular refractive index the rotation is therefore

$$R=\frac{\pi}{\lambda}(n_2-n_1).$$

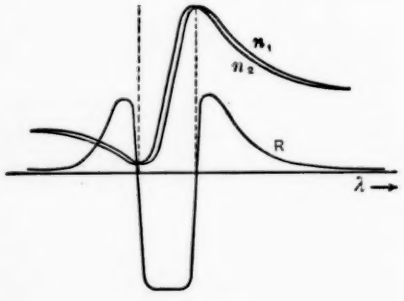

Fig. 11.

The figure shows the essential relation existing between difference in velocity (or difference in index) and rotation, independently of any theory of the mechanism of rotation. The ordinates of the

[1] Wied. Ann., 24, 127, 1885.
[2] Phil. Mag. (6), 1, 464, 1901.

rotation curve have been plotted proportional to the difference in the ordinates of the index curves. Now n_2 and n_1 are the same functions of λ and may be made congruent by changing λ by a constant. Call this constant a. Then if $n_1 = f(\lambda)$, $n_2 = f(\lambda + a)$. On the diagram, a is seen to be the distance between the maxima (or minima) of n_1 and n_2. Hence $n_2 - n_1 = f(\lambda + a) - f(\lambda)$. Expand this in a Taylor series and substitute in the above formula for the rotation and we have at once the rotatory dispersion formula

$$R = \frac{\pi}{\lambda}\left(a \cdot f'(\lambda) + \frac{a^2}{2} \cdot f''(\lambda) + \cdots\right).$$

leaving the molecular (or magnetic) constant a to be determined by experiment. Using but the first term of this expression, we have the familiar rotary dispersion formula

$$R = \frac{k}{\lambda} \cdot \frac{dn}{d\lambda}.$$

It is remarkable that by the above formula the rotation is proportional to the tangent of the angle of slope of the index curve and the figure shows this property very clearly. If the wave velocities are complex quantities, as they are when the absorption is finite, the rotation is not so simply expressed. This case will be discussed later with the mathematical theory of rotation.

The mechanism of structural rotation is perhaps the simplest of all three kinds. That a crystal possess rotatory power it is sufficient that its dielectric elasticity be different in three different directions and that it possess the dissymmetry above mentioned. Analytically this condition of dissymmetry says that it must never be possible so to choose our coördinate system that our equations of propagation shall remain unchanged on reversal of the direction of all three coördinate axes. Such dissymmetry can be introduced by the addition of a differential quotient of uneven order to the second order term in the ordinary wave equation. Boussinesq, v. Lang, Briot, Sarrau and Drude use the differential coefficients of the first order, MacCullagh and Larmor use those of the third order. Magnetic rotation is expressed in a similar way by introducing uneven differential coefficients with regard to the time instead of a coördinate.

The mechanism of the rotation of fluids must be quite different from that of crystals. It is not sufficient to assume that the molecular elasticity, like crystalline elasticity, is different in different directions, since the light waves are so large in comparison to the molecular dimensions and the molecules of a fluid are not supposed to be so arranged that their effect would be cumulative. Nor would such intermolecular forces as the forces that would exist between the separate magnets of a freely suspended group give the molecules any selective effect on circularly polarized light waves. In any molecule except one of the very simplest type, the effect of a light wave on the charged particles of which it is composed would be to set them in motion in curved paths, but the resultant reaction of a large number of such molecules on a light wave would be zero so far as its direction of vibration was concerned. The one molecular property essential in affecting the azimuth of the wave which sets it vibrating appears to be that the charged particles of which it is composed tend to move in helical paths under the influence of a linear force. A crude analogue would be a corkscrew immersed in a fluid; a sudden impulse in the direction of its length would tend to rotate it as well as move it forward. In a medium composed of molecules having such properties, a right circularly polarized wave would travel with a different velocity from a left-handed circular wave, hence a plane polarized wave would be broken up into circular components and all the phenomena of rotation would occur. And the enantiomorphic dissymmetry above described is just what would give a molecule this rotatory property. This molecular characteristic will be the basis of the mathematical theory developed. The assumption of fixed, stable atomic orbits within the molecule is not sufficient to explain the observed phenomena of rotation as Larmor[1] has shown. Chiral properties depend essentially on *molecular* character.

THEORY OF ROTARY DISPERSION.

Instead of an equation stating that a linear force produces a solenoidal displacement of a charged particle, it is simpler to set up an equation which says that a solenoidal force is necessary to pro-

[1] Phil. Mag., 44, 503.

duce a linear displacement of the particle. Having constructed this equation of motion for a single particle, we solve it for the motion. The product of this displacement and the involved charge, summed up for all the different kinds of charged particles and summed again for all the charges of each kind in a unit volume, gives the total electric displacement. Substituting this electric displacement in the general Maxwell equations and taking the periodic solution for the motion, we have in the velocity of propagation of these waves the dispersion theory desired.

Now in general a charge e is acted on with a force eX by an electric force X. The displacement of the charge will be resisted by an inertia force, an elastic force and a frictional force. Hence

$$(1) \qquad eX = m\frac{\partial^2\xi}{\partial t^2} + \frac{4\pi e^2}{\theta}\xi + \varphi e^2\frac{\partial\xi}{\partial t},$$

where m is the mass to which the charge e is supposed to be attached and ξ its displacement. θ measures the ease with which the charge is displaced elastically, the factor e^2 indicating that it is independent of the sign of the charge. φe^2 is the frictional coefficient, also independent of the sign of the charge. Now when the molecules are of the rotating kind, a solenoidal force proportional to $\frac{\partial Y}{\partial z} - \frac{\partial Z}{\partial y}$ must be added to the linear force eX to produce the linear displacement ξ. We have then [1] as an extension of (1) to the motion of the charged particles composing a rotating molecule:

$$(2) \qquad e\left[X + \rho\left(\frac{\partial Y}{\partial z} - \frac{\partial Z}{\partial y}\right)\right] = m\frac{\partial^2\xi}{\partial t^2} + \frac{4\pi e^2}{\theta}\xi + \varphi e^2\frac{\partial\xi}{\partial t}.$$

If now there be N of these charged particles alike in unit volume, the current density for this kind is $u_1 = eN\,d\xi/dt$. Now we are dealing with substances (non metals) in which elastic forces exist and θ is finite. Hence motion of the charged particle will be periodic, say $\xi = Ae^{i\frac{t}{\tau}}$. Substituting in (2) and solving for $e\xi$, the electric displacement concerned in the motion of the particle, we get for the current density:

[1] Drude, assuming quite a different mechanism of rotation, arrives at a similar equation. (See his Lehrbuch der Optik, p. 370.) The simple and ingenious development here presented (equations 1 to 13) is, I believe, due almost entirely to him.

$$u_1 = \frac{\theta N}{4\pi\left(1 + i\frac{a}{\tau} - \frac{b}{\tau^2}\right)} \frac{\partial}{\partial t}\left[X + \rho\left(\frac{\partial Y}{\partial z} - \frac{\partial Z}{\partial y}\right)\right] \tag{3}$$

where

$$a \equiv \varphi\theta/4\pi \text{ and } b \equiv m\theta/4\pi e^2 \equiv \tau_1^2.$$

Hence the total current density arising from the displacements of all the charged particles of all kinds and of the free ether as well is:

$$u = \frac{1}{4\pi}\frac{\partial}{\partial t}\left[\varepsilon X + \rho\left(\frac{\partial Y}{\partial z} - \frac{\partial Z}{\partial y}\right)\right], \tag{4}$$

where ε is the general (average) dielectric constant of the charge imbedded ether. Hence comparing with (3), ε is seen to be

$$\varepsilon = 1 + \Sigma \frac{\theta_1 N_1}{1 + i\frac{a}{\tau} - \frac{b}{\tau^2}} \tag{5}$$

and

$$\rho = \Sigma \frac{\theta_1 \rho_1 N_1}{1 + i\frac{a}{\tau} - \frac{b}{\tau^2}}.$$

To get the equation for the propagation of a wave in a rotating medium, we have only to substitute the generalized current (4) in the fundamental Maxwell equations. We thus obtain three equations of the type:

$$\frac{1}{V^2}\frac{\partial^2}{\partial t^2}\left[\varepsilon X + \left(\rho\frac{\partial Y}{\partial z} - \frac{\partial Z}{\partial y}\right)\right] = \Delta X. \tag{6}$$

For a beam of light travelling along the Z axis we put

$$X = Ae^{\frac{i}{\tau}(t - pz)},\quad Y = Be^{\frac{i}{\tau}(t - pz)},\quad Z = 0, \tag{7}$$

where p is the reciprocal of the velocity of propagation and A and B are complex constants. Substituting in (6), we have to determine the velocity and amplitude and of the wave propagated the two equations:

$$\begin{aligned} \varepsilon A - \frac{i}{\tau}\rho p B &= Ap^2V^2, \\ \varepsilon B - \frac{i}{\tau}\rho p A &= Bp^2V^2. \end{aligned} \tag{8}$$

Replacing A by $A + iA'$ and B by $B + iB'$, we find equations (8) to be satisfied by the two sets of values:

$$(9)\qquad \begin{aligned} \varepsilon - p^2V^2 &= -p\rho/\tau, \quad A = B' \\ \varepsilon - p^2V^2 &= +p\rho/\tau, \quad B = A'. \end{aligned}$$

Hence the velocity of propagation is double-valued, its reciprocal being:

$$(10)\qquad p = \pm \frac{\rho}{2\tau V^2} + \frac{1}{V}\sqrt{\frac{\rho^2}{4\tau^2V^2} + \varepsilon}.$$

Call these two values p_1 and p_2. Substituting in (7) and taking real parts we have the two circular waves

$$(11)\qquad \begin{aligned} X_1 &= A\cos 2\pi/T(t - p_1z), \quad Y_1 = \quad A\sin 2\pi/T(t - p_1z), \\ X_2 &= A\cos 2\pi/T(t - p_2z), \quad Y_2 = -A\sin 2\pi/T(t - p_2z). \end{aligned}$$

When the rotation factor ρ is positive, the second (clockwise) wave is the faster of the two. The resultant of the two waves is

$$(12)\qquad \begin{aligned} X &= X_1 + X_2 = 2A\cos\frac{1}{\tau}\left(t - \frac{p_2 + p_1}{2}z\right)\cos\frac{1}{\tau}\frac{p_2 - p_1}{2}z, \\ Y &= Y_1 + Y_2 = 2A\cos\frac{1}{\tau}\left(t - \frac{p_2 + p_1}{2}z\right)\sin\frac{1}{\tau}\frac{p_2 - p_1}{2}z, \end{aligned}$$

which is a plane polarized wave whose azimuth changes steadily with Z. At any z, the azimuth of the plane of polarization is given by

$$Y/X = \tan\frac{1}{\tau}\frac{p_2 + p_1}{2}z.$$

Hence in traversing a path of length l, the plane of polarization is rotated through the angle (expressed in radians)

$$(13)\qquad R = \frac{l}{\tau}\frac{p_2 - p_1}{2} = \frac{l}{\tau}\cdot\frac{\rho}{2\tau V^2} = \frac{l}{\tau V}\frac{n_2 - n_1}{2}$$

where n is the refractive index for circularly polarized waves. These expressions are of the same general form as those previously arrived at from geometrical considerations, but we are now in a position to interpret the rotation constant and the circular refractive indices. The rotation constant itself, ρ is proportional to the num-

ber of charges per unit volume and hence in a manner to the optical density. It is roughly proportional to the elastic resistance to displacement and to the ratio of charge to mass (cf. equations 5 and 3). Writing (13*c*) in the form

$$R = \frac{l}{4\tau V n}(n_2^2 - n_1^2),$$

and getting n from (5) by writing $n^2(1 - i\varkappa)^2 = \varepsilon$, $\varkappa$ being the absorptive index for circularly polarized waves, we obtain for the rotation

$$R = \frac{l\tau}{4nV} \Sigma \frac{\theta_2(\tau^2 - \tau_2^2) - \theta_1(\tau^2 - \tau_1^2)}{(\tau^2 - \tau_1^2)^2 + a\tau^2}, \tag{14}$$

a function having the general form of that represented in Fig. 11.

Now the refractive index of the fluids investigated has not been determined and (14) is further inconvenient on account of there being so many constants to determine. A formula just as simple and not containing n may be obtained by substituting (5*b*) directly in (13*b*). Retaining only the real part

$$R = \frac{l}{2\lambda^2} \Sigma \frac{\theta_1\left(1 - \frac{\tau_1^2}{\tau^2}\right)}{\left(1 - \frac{\tau_1^2}{\tau^2}\right)^2 + \frac{a_1}{\tau^2}}.$$

Now the experimental work relates to a region in which τ is much greater than τ_1, hence neglecting the fourth power of that quantity in comparison with the second, we have finally the working formula

$$R = \frac{k_1(\lambda^2 - \lambda_1^2)}{\lambda^2(\lambda^2 - 2\lambda_1^2 + a_1)} + \frac{k_2(\lambda_2 - \lambda_2^2)}{\lambda^2(\lambda^2 - 2\lambda_2^2 + a_2)} + \cdots. \tag{15}$$

This formula fulfils all of the four conditions we have to impose upon it. Far from any region of absorption it expresses the inverse square law. Approaching a region of absorption, the denominator will approach the value zero more rapidly than the numerator and hence the rotation will increase much more rapidly than the inverse square of the wave-length. Within the region of absorption, for λ^2 between λ_0^2 and $2\lambda_0^2 - a$, the rotation is reversed in sign. It remains but to test the formula for the fourth requirement, namely that it closely represent the observed data using but one or two terms.

Using two terms, the observed and calculated curves would lie so near together as hardly to be distinguishable, since there would be six constants involved and the two curves would cross in at least six different places. However, using but a single term, the calculated curve lies very close to the observed. This single term test is the severest that could be applied, and the close agreement shows beyond question that the derived dispersion function is of the proper form. Since the absorption is not confined to a narrow band, we ought properly to take the sum of a number of terms having constants very nearly alike, but it may be shown mathematically that such a sum is very approximately equivalent to a single term with different constants. The constants determined for the single term formula are given below. Since differences of small quantities are involved, it is necessary to carry the calculations to seven figures.

	λ_0	λ_∞	a	k
Cane sugar.	189.6317	213.2	0.0287058	22.7886
Camphor.	276.0559	326.05	0.0461062	16.2606
Tartaric acid.	373.300	257.7	0.211035	3.5006
Potas. ant. tartrate.	254.6130	274.9	0.0540385	38.9839

The constant k gives the rotation in degrees instead of radians. λ_0 is the wave-length at which the dispersion curve would cross the wave-length axis if produced. λ_∞, where $\lambda_\infty^2 = 2\lambda_0^2 - a$, is the ordinate toward which the dispersion curve tends to approach asymptotically. Very roughly, the smaller λ_0, the straighter the curve. The larger the constant a, the greater the departure from the $1/\lambda^2$ law.

The curves calculated with the above constants are shown dotted in the figures. The characteristic (one term) form of the dispersion curve is seen to be midway between the extreme types observed, less nearly the arc of a circle than potassium antimony tartrate and not so much the shape of a sled runner as the sugar and camphor curves.

In conclusion I desire to express my warmest thanks to Professor Nichols, of the Physics Department, and to Professor Orndorff and Mr. Teeple, of the Department of Chemistry, for their unfailing courtesy in placing at my disposal the resources of their respective departments.

CORNELL UNIVERSITY,
April, 1903.

THE PRESSURE DUE TO RADIATION.

(SECOND PAPER.)

BY E. F. NICHOLS AND G. F. HULL.

IN a preliminary communication to the American Physical Society, meeting with Section B of the American Association for the Advancement of Science at Denver, August, 1901, the writers presented the results of observations they had made on the pressure due to radiation. An abstract of that paper appeared in Science, October, 1901, and the full paper in this journal in November of the same year. In the present paper the writers wish to present a more complete historical statement than appeared in the preliminary paper, alterations which were made in the earlier methods, and more recent observations and data under conditions permitting greater accuracy in measurement.

As early as 1619 Kepler[1] announced his belief that the solar repulsion of the finely divided matter of comets' tails was due to the outward pressure of light. On the corpuscular theory of light Newton[2] considered Kepler's idea as plausible enough, but he was of the opinion that the phenomenon was analogous to the rising of smoke in our own atmosphere. In the first half of the eighteenth century DeMairan and DuFay[3] contrived elaborate experiments to test this pressure of light theory in the laboratory, but, because of the disturbing action of the gases surrounding the illuminated bodies employed in the measurements, they obtained wholly confusing and contradictory results. Later in the same ceutury the Rev. A. Ben-

[1] De Mairan, Traité physique et historique de l'Aurore Boréale (seconde edition), pp. 357–358, Paris, 1754.

[2] Isaaci Newtoni, Opera quae Existent Omnia. Samuel Horsley, LL.D., R.S.S., Tom. III., pag. 156, Londinium, 1782.

[3] De Marian, l. c., p. 371. This treatise contains also the accounts of still earlier experiments by Hartsoeker, p. 368, and Homberg, p. 369. The later experiments are of more historic than intrinsic interest.

net[1] performed further experiments but could find no repulsive force not traceable to convection currents in the gas surrounding the body upon which the light was projected, due in his opinion to the heating effect of the rays. Finding no pressure due to radiation, he made the following unique suggestion in support of the wave theory of light: "Perhaps sensible heat and light may not be caused by the influx or rectilinear projection of fine particles, but by the vibrations made in the universally diffused caloric or matter of heat or fluid of light. I think modern discoveries, especially those of electricity, favor the latter hypothesis." In the meantime Euler,[2] accepting Kepler's theory attributing the phenomenon of comets' tails to light pressure, had hastened to the support of the wave theory by showing theoretically that a longitudinal wave motion might produce a pressure in the direction of its propagation upon a body which checked its progress. In 1825 Fresnel[3] made a series of experiments, but arrived at no more definite conclusion than that the repulsive and attractive forces observed were not of magnetic nor electric origin.

Crookes[4] believed in 1873 that he had found the true radiation pressure in his newly invented radiometer and cautiously suggested that his experiments might have some bearing on the prevailing theory of the nature of light. Crookes' later experiments and Zöllner's[5] measurements of radiometric repulsions showed that the radiometric forces were in some cases 100,000 times greater than the light pressure forces with which they had been temporarily confused. Zöllner's experiments are among the most ingenious ever tried in this field of work, and he missed the discovery of the true radiation pressure by only the narrowest margin. An excellent bibliography of the whole radiometric literature is given by Graetz,[6] and an account of some of the older experiments not mentioned above is given by Crookes.[7]

[1] A. Bennet, Phil. Trans., p. 81, 1792.
[2] L. Euler, Histoire de l'Academie Royale de Berlin (2), p. 121, 1746.
[3] A. Fresnel, Ann. Chem. et Phys., XXIX., pp. 57, 107, 1825.
[4] W. Crookes, Phil. Trans., p. 501, 1873.
[5] F. Zöllner, Pogg. Ann., CLX., pp. 156, 296, 459, 1877.
[6] L. Graetz, Winkelmann's Handbuch der Physik, 2 b, p. 262. Breslau, 1896.
[7] W. Crookes, l. c., p. 501.

In 1873 Maxwell,[1] on the basis of the electromagnetic theory, showed that if light were an electromagnetic phenomenon, pressure should result from the absorption or reflection of a beam of light. After a discussion of the equations involved, he says: "Hence in a medium in which waves are propagated there is a pressure in the direction normal to the waves and numerically equal to the energy in unit volume." Maxwell computed the pressure exerted by the sun on the illuminated surface of the earth and added: "It is probable that a much greater energy of radiation might be obtained by means of the concentrated rays from an electric lamp. Such rays falling on a thin metallic disc, delicately suspended in a vacuum, might perhaps produce an observable mechanical effect."

Apparently independent of Maxwell, Bartoli[2] announced in 1876 that the Second Law of Thermodynamics required the existence of a pressure due to radiation numerically equal in amount to that derived by Maxwell. Bartoli's reasoning holds for all forms of energy streams in space and is of more general application than Maxwell's equations. Bartoli contrived elaborate experiments to verify this theory, but was balked in the search, as all before him had been, by the complicated character of the gas action which he found no way of eliminating from his experiments.

After Bartoli's work the subject was dealt with theoretically by Boltzmann,[3] Galitzine,[4] Guillaume,[5] Heaviside,[6] and more recently by Goldhammer.[7] Fitzgerald,[8] Lebedew,[9] and Hull[10] have discussed the bearing of radiation pressure upon the Newtonian law of gravitation with special reference to the repulsion of comets' tails by the sun.

[1] J. C. Maxwell, A Treatise on Electricity and Magnetism (1st edition), II., p. 391, Oxford, 1873.

[2] A. Bartoli, Sopra i movementi prodotti della luce e dal calorie, Florence, Le Monnier (1876), also Nuovo Cimento, XV., p. 193, 1884.

[3] L. Boltzmann, Wied. Ann., XXII., pp. 31, 291, 1884.

[4] B. Galitzine, Wied. Ann., XLVII., p. 479, 1892.

[5] Ch. Ed. Guillaume, Arch. de Gen. (3), XXXI., p. 121, 1894.

[6] O. Heaviside, Electromagnetic Theory, I., p. 334, London, 1893.

[7] D. A. Goldhammer, Ann. Phys., IV., p. 834, 1901.

[8] G. F. Fitzgerald, Proc. Roy. Soc. Dub., 1884.

[9] P. Lebedew, Wied. Ann., XLV., p. 292, 1892. Astrophys. Jour., XIV., p. 155, 1902.

[10] G. F. Hull, Trans. Astron. Soc. Toronto, p. 123, 1901.

The theory of radiation pressure combined with the known properties of negative electrons has recently been more or less speculatively applied by Arrhenius [1] to the explanation of many cosmical and terrestrial phenomena among which the following may be mentioned: The solar corona, zodiacal light, gegenschein, comets, origin of cometary and meteoric material in space, the emission of gaseous nebulæ, the peculiar changes observed in the nebula surrounding Nova Persei, the northern light, the variations in atmospheric electricity and terrestrial magnetism and in the barometric pressure. Swartzschild [2] computed from radiation pressure on small spherical conductors the size of bodies of unit density for which the ratio of radiation pressure to gravitational attraction would be a maximum.

Before the Congrès International de Physique in 1900, Professor Lebedew,[3] of the University of Moscow, described an arrangement of apparatus which he was using at that time for the measurement of light pressure. He summarizes the results already obtained as follows: "Les résultats des mesures que j'ai faites jusqu'ici peuvent se résumer ainsi: L'expérience montre qu'un faisceau lumineux incident exerce sur les surfaces planes absorbantes et réfléchissantes des pressions qui, aux erreurs près d'observation, sont égales aux valeurs calculées par Maxwell et Bartoli." No estimate of the "errors of observation" was given in the paper nor other numerical data. Unfortunately the proceedings of the Paris Congress did not reach the writers nor any intimation of the methods or results of Professor Lebedew's work until after the publication of their own preliminary experiments.

In the preliminary paper already referred to, the writers presented the results they had obtained by measurement of radiation pressure at various gas pressures. The main arguments underlying the method of measurement of the radiation pressure follow.

In the experiments of earlier investigators every approach to the experimental solution of the problem of radiation pressure had been

[1] S. A. Arrhenius, Lehrbuch der Cosmischen Physik, Leipzig, 1903, pp. 149–158, 200–208, 226, 920–925.

[2] K. Swartschild, Kgl. Bayer. Akademie d'Wissenschaften, XXXI., 293, 1901.

[3] P. Lebedew, Rapports présentés au Congrès International de Physique (2), p. 133, Paris, 1900.

balked by the disturbing action of gases which it is impossible to remove entirely from the space surrounding the body upon which the radiation falls. The forces of attraction or repulsion, due to the action of gas molecules, are functions, first, of the temperature difference between the body and its surroundings, caused by the absorption by the body of a portion of the rays which fall upon it; and second, of the pressure of the gas surrounding the illuminated body. In the particular form of apparatus used in the present study the latter function appears very complicated, and certain peculiarities of the gas action remain inexplicable upon the basis of any simple group of assumptions which the writers have so far been able to make.

Since we can neither do away entirely with the gas nor calculate its effect under varying conditions, the only hopeful approach which remains is to devise apparatus and methods of observation which will reduce the errors due to gas action to a minimum. The following considerations led to a method by which the elimination of the gas action was practically accomplished in the present experiments.

1. The surfaces which receive the radiation, the pressure of which is to be measured, should be as perfect reflectors as possible. This will reduce the gas action by making the rise of temperature due to absorption small, while the radiation pressure will be increased; the theory requiring that a beam, totally reflected, exert twice the pressure of an equal beam, completely absorbed.

2. By studying the action of a beam of constant intensity upon the same surface surrounded by air at different pressures, certain pressures may be found where the gas action is less than at others.

3. The apparatus — some sort of torsion balance — should carry two surfaces symmetrically placed with reference to the rotation axis, and the surfaces on the two arms should be as nearly equal as possibly in every respect. The surfaces or vanes should be so constructed that if the forces due to gas action (whether suction or pressure on the warmer surface) and radiation pressure have the same sign in one case, a reversal of the suspension should reverse the gas action and bring the two forces into opposition. In this way a mean of the forces on the two faces of the suspension should be, in part at least, free from gas action.

4. Radiation pressure, from its nature, must reach its maximum value instantly, while observation has shown that gas action begins at zero and increases with length of exposure, rising rapidly at first, then more slowly to its maximum effect, which, in many of the cases observed, was not reached until the exposure had lasted from two and a half to three minutes. For large gas pressures, an even longer exposure was necessary to reach stationary conditions. The gas action may be thus still further reduced by a ballistic or semi-ballistic method of measurement.

Ballistic observations of radiation pressure were made at air pressures ranging from 96.3 mm. to 0.06 mm. of mercury. The average radiation pressure of the standard beam was found to be 1.05×10^{-4} dyne with a probable error of 6 per cent. To compare this value of the pressure with the theoretical value as given by the Maxwell-Bartoli formula,

$$p = \frac{E(1+\rho)}{V},$$

it was necessary to measure E, the energy of the beam. This was done by measuring the resistance of a platinum disc P (Fig. 1) when it was exposed to the radiation and by determining the heat generated in P by an electrical current across AB when the resistance of P was equal to the value it possessed when it was exposed to the beam. Using the value of the energy thus obtained and 0.92[1] as the reflection coefficient of the silver surfaces, it was found that the pressure directly observed was about 20 per cent. lower than that computed from Maxwell's formula.

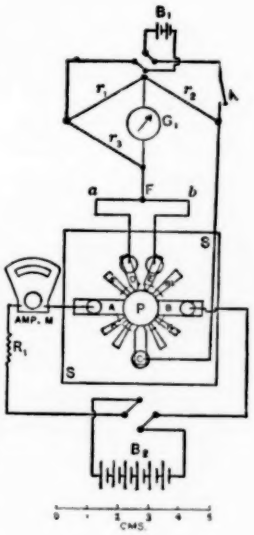

Fig. 1.

After the publication of the paper it was discovered that in dissolving the silver from the platinum when the bolometer was made, the acid had eaten away the silver from the strips A and B for a

[1] This value was obtained from the measurements of Langley, Rubens, Nichols and Paschen for the assumed mean wave-length of the energy of the beam.

distance of nearly a millimeter under the asphalt coating. The resistance 0.278 ohm given for the disc was thus too high. It was impossible to redetermine the resistance by the direct method because of an accident to the bolometer by which the disc was nearly severed from the strip B. The disc was therefore carefully torn away from its supports, mounted on a glass plate and cut on a dividing engine into strips, 1 and 2 mm. wide, parallel to AB. The resistance along these strips was measured by the fall of potential method. The resistance was found to vary slightly in different parts of the disc due to lack of uniformity in the thickness of the metal. After many measurements, an average value was reached and the resistance of the disc computed theoretically as follows: —

The resistance of a conducting sheet of infinite extent, when the current enters and leaves the sheet by electrodes[1] of relatively great conductivity, is $\sigma/4\pi C$, where σ is the resistance of any square of the sheet, and C is the electrostatic capacity of the two electrodes. If the electrodes are cylinders, the lines of flow are circles orthogonal to them. When the sheet, in place of being infinite, is bounded by one of these circular lines of flow, the resistance is $\sigma/2\pi C$. In particular if the electrodes of radii r are on a diameter of this circular sheet of radius R, then the resistance can be shown to be

$$\frac{\sigma}{\pi}\log_e \frac{2r^2 + R^2 + R\sqrt{4r^2 + R^2}}{2r^2}.$$

Assuming for the moment that the leading-in strips of the bolometer (Fig. 1) were of great conductivity compared to that of the thin platinum sheet and that they terminated in circular arcs orthogonal to this circular sheet, the resistance would be, giving to r the value of 2.79 mm. and to R the value 11.25 mm., $0.922 \times \sigma$. But the leading-in strips terminated on the boundary of the large circle. The resistance was therefore altered by two facts — the lines of flow were changed and the distance between the electrodes was increased. The latter is the important item. It is necessary therefore to find approximately the resistance of these gibbous portions of the large disc previously considered as electrodes. This may be done by estimating the area of these parts and by considering the average

[1] J. J. Thomson, Electricity and Magnetism, 2d edition, p. 314, Cambridge, 1897.

equipotential line as midway between the cord and arc of the cylindrical electrode. It results that the amount to be added on account of this calculation is $0.471 \times \sigma$. Hence the resistance between the electrodes is now $(0.922 + 0.471)\,\sigma = 1.393 \times \sigma$. The value of σ as found by the fall of potential method was 0.148 at 19° C. When corrected for the temperature of the disc exposed to the lamp, σ becomes 0.160. Hence the resistance of the disc when hot was $1.393 \times 0.160 = 0.221$ ohm.[1] Substituting this computed value of the resistance in place of the one used, the energy of the standard beam becomes $0.221 \times 0.75 \times 10^7$ ergs-seconds and

$$p = \frac{1.92 \times 0.221 \times 0.75 \times 10^7}{3 \times 10^{10}} = 1.05 \times 10^{-4} \text{ dynes.}$$

This result is in accidental agreement with the observed pressure. If necessary corrections, determined by later experiment, had been applied, the difference between the observed pressure and the pressure computed from the energy measurements would have been about three per cent. Moreover the probable error of the final result was roughly double this amount.

In the November number of the Annalen der Physik for 1901 Professor Lebedew[2] published the results of a more varied series of measurements of radiation pressure than the early measurements of the present writers. The principal difference between the methods employed by him and by the writers for determining the pressure was that he used very thin metallic vanes surrounded by gas at extremely low pressures, thus following Maxwell's suggestion literally, while the writers used silvered glass vanes and worked at large gas pressures for which the gas action had been carefully and exhaustively studied and found to be negligibly small for short exposures. From our knowledge of the variation of gas action in different vacua, we feel sure that our method would not have been successful in high vacua because of the relatively large gas action. Professor Lebedew's own results, with blackened vanes of lower heat conductivity, show that his success in eliminating gas distur-

[1] The resistance of a trial disc was measured experimentally with the result that the experimental value differed from the theoretical by about one per cent.

[2] P. Lebedew, Ann. Phys., VI., 433, 1901.

bance was due to the high heat conductivity of thin vanes rather than to the high vacua employed.

Professor Lebedew's[1] estimate of the accuracy of his work is such as to admit of possible errors of twenty per cent. in his final results. An analysis of Professor Lebedew's paper and comparison with our preliminary experiments seems to show that his accidental errors were larger than ours, but through the undiscovered false resistance in the bolometer our final results were somewhat further from the theory than his. Either of the above researches would have been sufficient to establish the *existence* of a pressure due to radiation, but neither research offered, in our judgment, a satisfactory *quantitative* confirmation of the Maxwell-Bartoli theory.

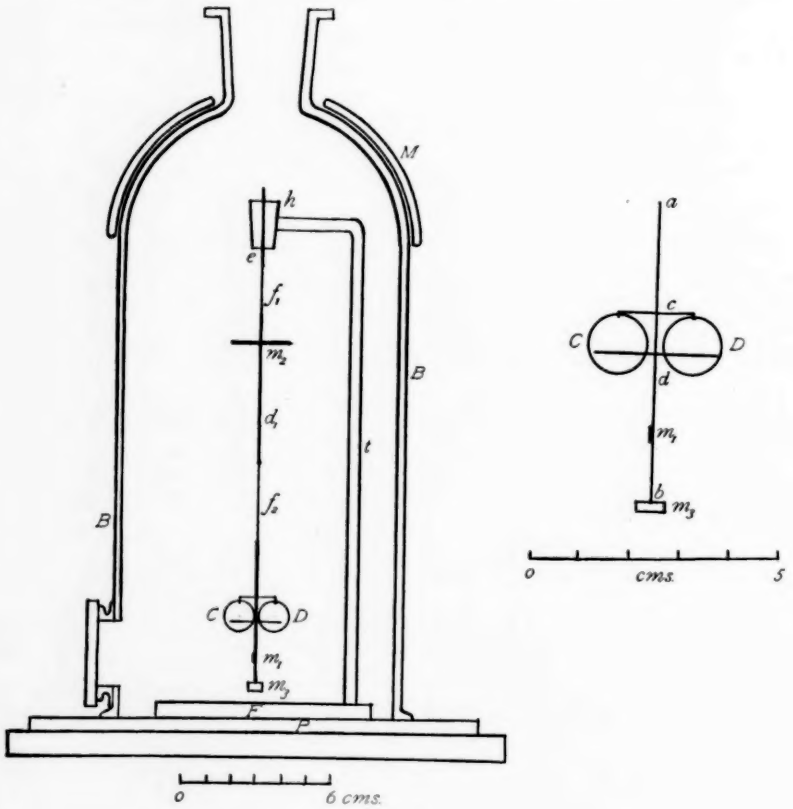

Fig. 2.

[1] P. Lebedew, Ann. Phys., VI., 457, 1901.

Later Pressure Measurements.

Description of Apparatus.—The Torsion Balance.

The form of suspension of the torsion balance, used to measure radiation pressure in the present study, is seen in Fig. 2 and is described in detail in our preliminary paper. Its essential parts are a fine glass rod *ab* to which was attached a weight m_3, a scale mirror m_1, a cross arm *c* holding the mirrors *C* and *D*; a quartz fiber f_2; a glass rod d_1 to which was attached a small magnet m_2 and a silk fiber f_1. The cover glasses *C* and *D* which served as vanes were silvered and brilliantly polished and were so hung on the small hooks that both silver or both glass faces were presented to the light. A semicircular magnet *M*, fitted to the vertical curvature of the bell-jar, was used to direct the suspended magnet m_2 and thus to control the zero position of the torsion balance. By turning *M* through 180°, the opposite faces of the vanes *C* and *D* could be presented to the light.

The Arrangement of Apparatus.

A horizontal section of the apparatus through the axis of the light beam is shown in Fig. 3. An image of the aperture d_3, very brightly illuminated by the white-hot carbon S_1 and the lenses L_1 and L_2, was formed by the lens L_3 in the plane B_1. A shutter S_2 controlled by a magnetic escapement operated by the seconds' contact of a standard clock permitted the exposure of the beam upon the vanes for any whole number of seconds. The stops S_3 and S_4 were so arranged that when the lens L_3 was against either stop the image of d_3 was central upon one of the vanes. A glass plate inclined at an angle of 45° to the axis of the beam and a lens L_5 gave an image of d_3 upon one arm of the bolometer at *R*. This bolometer was of sheet platinum 0.001 mm. thick rolled in silver. The strip was cut out in the form shown in Fig. 4, and mounted on a thin sheet of slate *S*. Two windows had been cut in the slate behind the strips at *AB*, *CD* where the silver had been removed, leaving the thin platinum. The platinum surfaces were blackened by Kurlbaum's process. The image from L_5 (Fig. 3) fell at *D*. The silver ends between *A* and *C* were connected with *E* and *F* respec-

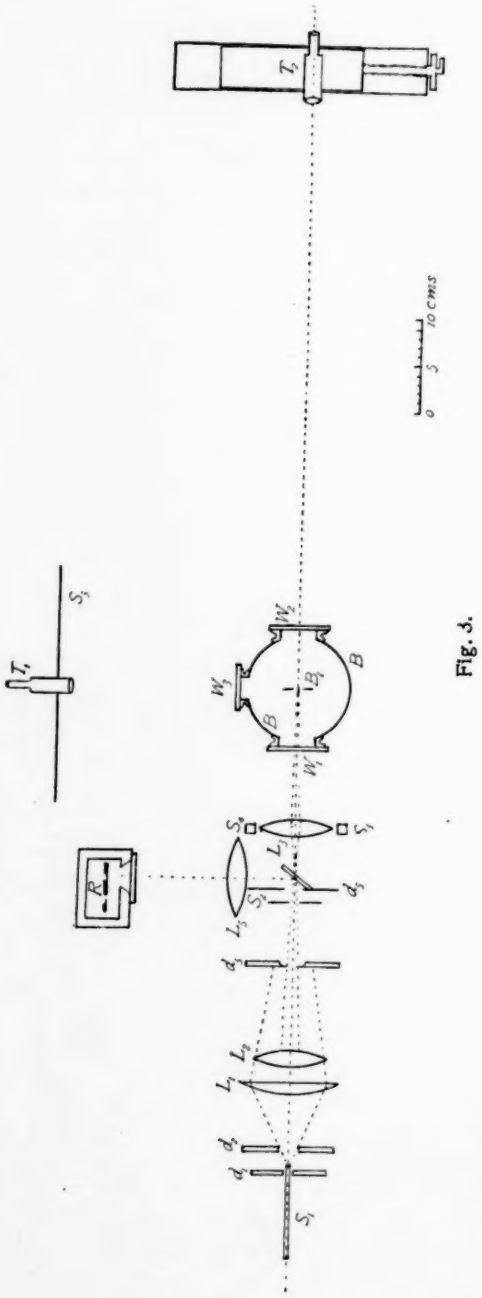

Fig. 3.

various parts about the axis of rotation.

tively. On the heavy wire *EF* a sliding contact *c* served to balance the bridge, all four arms of which are shown in the figure.

METHODS OF OBSERVATION.

The observations leading to the results given later were of three different kinds: (1) The calibration of the torsion balance; (2) the measurement of the pressure of radiation of the law in terms of the constant of the balance; and (3) the measurement of the energy of the same beam in erg-seconds by the rate of temperature rise of a blackened silver disc, of known mass and specific heat.

1. The determination of the constant of the torsion balance was made by removing the vanes *C* and *D* and accurately measuring the period of vibration. Its moment of inertia was easily computed from the masses and distribution of the The moment of torsion

for 1 mm. deflection on a scale 105 cm. distant was 0.363×10^{-5} dyne $\times$ cm. This value divided by one half the distance between the centers of the light spots on the two vanes gave the force in dynes per scale division deflection. As the light spots were circles

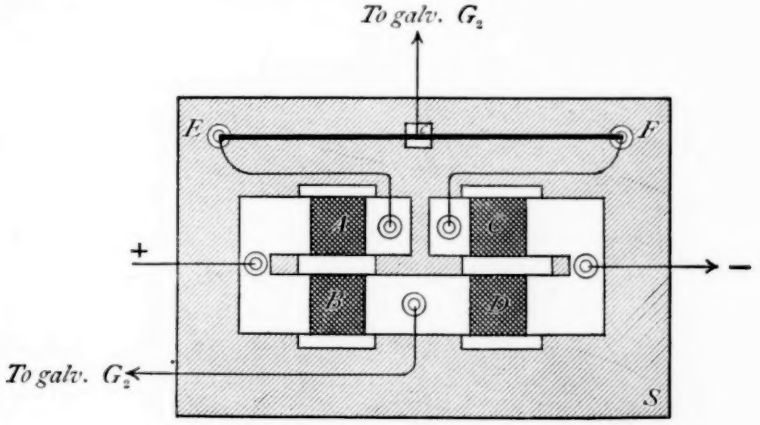

Fig. 4.

11.25 mm. in diameter the area of the image was very nearly 1 $(\text{cm.})^2$, hence the above procedure gave roughly the pressure in dynes per square centimeter.

2. In the measurements of radiation pressure, it was easier to refer the intensity of the beam at each exposure to some arbitrary standard which could be kept constant than to try to hold the lamp as steady as would otherwise have been necessary. For this purpose, the bolometer at R (Fig. 3) was introduced, and simultaneous observations were made of the relative intensity of the reflected beam by the deflection of the galvanometer G_2, and the pressure due to the transmitted beam by the deflection of the torsion balance. The actual deflection of the balance was then reduced to a deflection corresponding to a galvanometer deflection of 100 scale divisions. The galvanometer sensitiveness was carefully tested at the beginning and end of each evening's work. All observations of pressure were thus reduced to the pressure due to a beam of fixed intensity.

At each series of radiation pressure measurements, two sets of observations were made. In one of these sets, static conditions were observed, and in the other, the deflections of the balance due

to short exposures were measured. In the static observations, each vane of the balance was exposed in turn to the beam from the lamp, the exposures lasting until the turning points of the swings showed that stationary conditions had been reached. The moment of pressure of radiation and gas action combined would thus be equal to the product of the static deflection and the constant of the balance. The torsion system was then turned through 180° by rotating the outside magnet, and similar observations were made on the reverse side of the vanes. All turning points of the swinging balance in these observations were recorded. From the data thus obtained the resultant of the combined radiation and gas forces could be determined for the time of every turning point. Every value was divided by the deflection at standard sensitiveness of the galvanometer G_2 read at the same time and was thus reduced to a standard lamp. Results thus obtained, together with the ballistic measurements, showed the direction and extent of the gas action as well as its variation with length of exposure.

The reasons for reversing the suspension follow: The beam from the lamp, before reaching the balance, passed through three thick glass lenses and two glass plates. All wave-lengths destructively absorbed by the glass were thus sifted out of the beam by the time it reached the balance vanes. The silver coatings on the vanes absorbed therefore more than the glass. The radiation pressure was always away from the source irrespective of the way the vanes were turned, while the gas action would be exerted mainly on the silvered sides of the vanes.

At the close of the pressure and energy measurements, when the reflecting power of the silver faces of the vanes was compared with that of the glass-silver faces, the reflection from the silver faces was found very much higher than that for the glass faces backed by silver. This result was the more surprising because the absorption of the unsilvered vanes was found by measurement to be negligibly small.[1] This unexpected difference in reflecting power of the two faces of the mirrors prevented the elimination of the gas action, by the method described, from being as complete as had

[1] Lord Rayleigh records a similar difference between the reflection from air-silver and glass-silver surfaces. Scientific Papers, Cambridge, II., pp. 538–539, 1900.

been hoped for. But by choosing a gas pressure where the gas action after long exposure is small, the whole gas effect during the time of a ballistic exposure may be so reduced as to be of little consequence in any case.

By exposing each of the vanes in turn and by reversing the suspension and averaging results, nearly all errors due to lack of symmetry in the balance or in the position of the light images with reference to the rotation axis, or errors due to lack of uniformity in the distribution of intensity in different parts of the image, could be eliminated.

The changing character of the gas action, both with time of exposure and gas pressure surrounding the balance vanes is well illustrated in eight series of static observations in which the glass faces of both vanes were exposed.[1] The results obtained on the two vanes were averaged and plotted as curves in Fig. 5, where static deflections due to combined radiation pressure and gas action are shown as ordinates and duration of exposure, in seconds, as abscissæ.[2] A horizontal line through the diagram gives the mean value of the moment of radiation pressure computed from the data in Table II. Decrease of the deflection with time indicates gas repulsion on the warmed silver faces and increase in deflection, gas suction. It will be seen from the curves that beginning at a gas pressure of 66 mm. of mercury, the gas action was repulsion changing to suction in passing from 19.8 to 11.2 mm. In the last two cases the total gas action is small. For lower pressures the suction increases to 0.05 mm. At a gas pressure of 0.02 mm. the gas action is again a strong repulsion.

The curves indicate the existence of two gas pressures, at which the gas action in our arrangement of apparatus should be zero, one between 19.8 and 11.2 mm. and the other between 0.05 and 0.02 mm.[3] The former region was chosen for the ballistic measurements

[1] Observations were also made on the silver faces, but the gas action when the glass faces were exposed was considerably greater that for the silver faces, so the least favorable case is shown.

[2] Ordinates of the curves are proportional to moments.

[3] Crookes in his work with the radiometer discovered certain gas pressures for which the combined gas and radiation forces neutralized, but as he did not discriminate between forces due to radiation and gas forces his results were apparently capricious and his reasoning somewhat confused. See Phil. Trans., p. 519, 1875.

and nearly all of the observations were made at a gas pressure of approximately 16 mm. Even for the two pressures where the decrease in the static deflection was most rapid, *i. e.*, at gas pres-

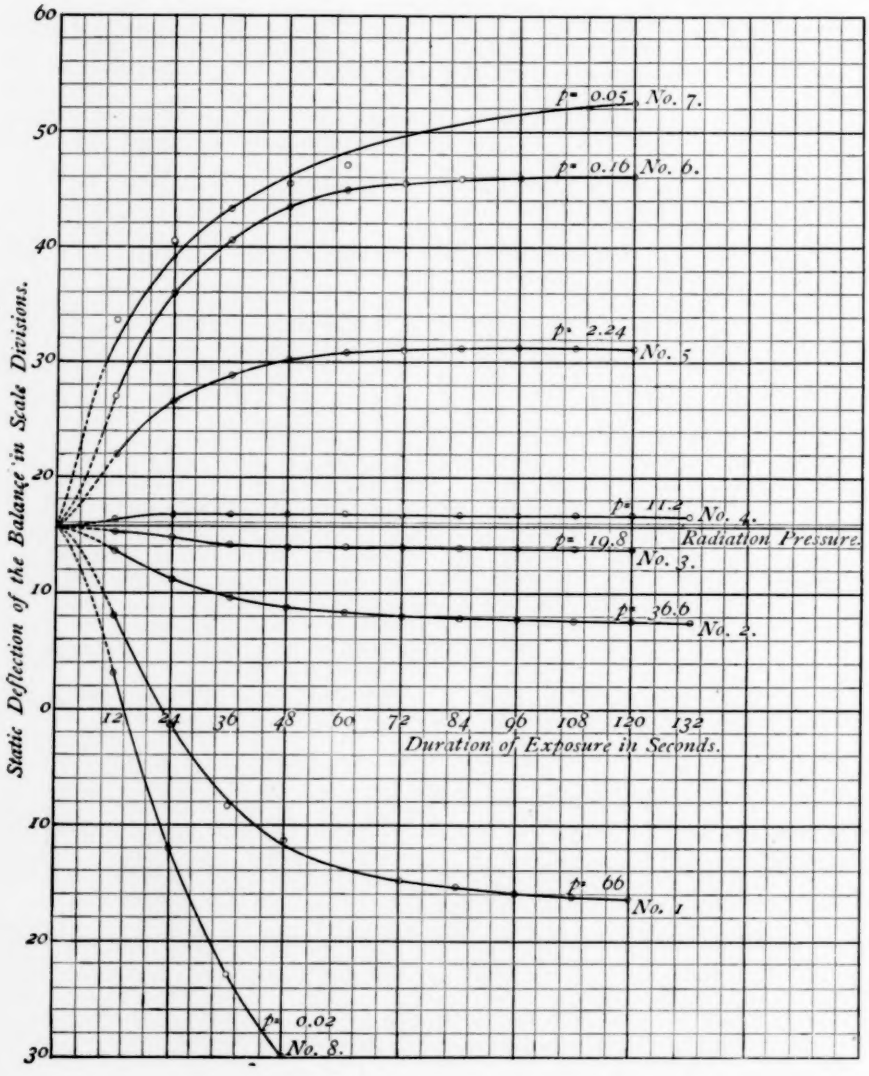

Fig. 5.

sures of 66 and 0.02 mm., the first throw was always in the direction of radiation pressure. The gas action is strongly influenced

by very slight changes in the inclination of the plane of the vanes to the vertical and also by any object introduced under the bell-jar anywhere near the vanes. For instance, a very considerable effect was observed when a small vessel of phosphoric anhydride was placed under the jar behind the vanes, though the nearest wall of the vessel was separated from the vanes by a distance of at least 3 cm.

During the observations, the polished silver coatings on the vanes deteriorated rapidly; new coatings rarely lasted for more than two evenings' work. As the balance had to be removed and the mirrors taken from the hooks, silvered, polished, and replaced a great number of times during the entire series of measurements, although great care was taken in setting the plane of the vanes vertical, it is not likely that precisely the same conditions for gas action were ever repeated. The principal value of the static results was in indicating favorable gas pressures for work, rather than affording quantitative estimates of the gas action in short exposures. The dotted parts of the curves are not based on results of observation and might perhaps have been omitted without loss.

It was plain, therefore, that further elimination of the gas action must be sought in exposures so short that the gas action would not have time to reach more than a small fraction of its stationary value. This led to the method of ballistic observations.

The Ballistic Observations.

In passing from the static to the ballistic observations it must always be possible to compute the static equivalent of the ballistic swings. Furthermore, the exposures should be made as short as possible without reducing the size of the swing below a value which can be accurately measured.

If the exposure lasts for one half the period of the balance, the deflection, if the gas action be small and the damping zero, is equal to 2θ, where θ is the angle at which the torsion of the fiber will balance the moment produced by the radiation pressure. If the duration of the exposure be one quarter of the period of the balance, the angle of deflection is $\theta\sqrt{2}$. The deflection is thus reduced by 30 per cent., but the effect of the gas action is reduced in greater

proportion. It was decided therefore to expose, for six seconds, one quarter of the balance period. Neglecting the gas action, the equation[1] of motion of the balance is given by

$$\varkappa \frac{\partial^2\theta}{\partial t^2} + 2\varepsilon \frac{\partial\theta}{\partial t} = -G\theta + L$$

where $\varkappa$ = the moment of inertia of the torsion balance,

ε = the damping constant,

G = the moment of torsion of the fiber for $\theta = 1$ radian,

and L = the moment of the radiation force.

The solution of this equation is

$$\theta = \frac{L}{G}\left\{1 - e^{-\frac{\varepsilon}{\varkappa}t}\cos\sqrt{\frac{G}{\varkappa} - \frac{\varepsilon^2}{\varkappa^2}}\,t\right\}$$

$$= \frac{L}{G}\left\{1 - e^{-\frac{\varepsilon}{\varkappa}t}\cos\ 2\pi\frac{t}{T}\right\} \qquad (1)$$

the constants of integration having been determined from the condition that

$$\theta = \frac{\partial\theta}{\partial t} = 0 \text{ when } t = 0.$$

When

$$t = \frac{T}{4}, \quad \theta = \frac{L}{G}$$

and

$$\frac{\partial\theta}{\partial t} = \frac{L}{G}\left(\frac{\varepsilon}{\varkappa}e^{-\frac{\varepsilon}{\varkappa}t}\cos\ 2\pi\frac{t}{T} + e^{-\frac{\varepsilon}{\varkappa}t}\frac{2\pi}{T}\sin\ 2\pi\frac{t}{T}\right). \qquad (2)$$

The light being cut off when $t = T/4$, the equation of motion becomes

$$\varkappa \frac{\partial^2\theta}{\partial t^2} + 2\varepsilon \frac{\partial\theta}{\partial t} = -G\theta \qquad (3)$$

the solution of which is $\theta = Ae^{-\frac{\varepsilon}{\varkappa}t}\cos\left(2\pi\frac{t}{T} + a\right)$ where A and a

[1] We are justified in using quantitatively this equation, containing a damping term proportional to the velocity, because the amplitudes of the successive swings of the torsion balance, when no energy fell upon the vanes, were found experimentally to follow accurately the exponential law.

can be determined by the conditions imposed by equation (2). Neglecting very small quantities, the value of the amplitude A is expressed by the equation

$$A = \frac{L}{G}\left\{1 + r + \frac{2}{\pi}r^{\frac{1}{2}}\log\left(\frac{1}{r}\right)\right\}^{\frac{1}{2}}, \tag{4}$$

where r is the ratio of successive amplitudes of the damped vibrations. If $r = 1$, that is if the motion is undamped, $A = \sqrt{2}L/G$. In the partial vacuum used in the experiments (16 mm. of mercury, a value chosen from the curves in Fig. 5), r was found to be equal to 0.783 ; consequently

$$A = 1.357\,\frac{L}{G}. \tag{5}$$

From this it is seen that the total angle of deflection of the torsion balance in the ballistic measurements is equal to 1.357 times the angle at which the moment of the torsion of the fiber balances the moment of the radiation pressure.

The duration of exposure was always six seconds without appreciable error, but the period of the balance on account of slight accidental shifting of small additional masses upon the counterpoise weight m_3 (Fig. 2), differed from twenty-four seconds sometimes by one per cent. It is necessary therefore to find the error in the deflection due to this variation in the period. This is done by making $t = T/4 + \delta$ in equation (2) and in introducing the new conditions in equation (3). But it is simpler and sufficiently accurate to assume the motion as undamped. For this condition, the amplitude

$$A = \frac{L}{G}\left\{2 + 2\sin 2\pi\frac{\delta}{T}\right\}^{\frac{1}{2}} = \sqrt{2}\,\frac{L}{G}\left(1 + \pi\frac{\delta}{T}\right) \text{ nearly.}$$

For $T = 23.75$ seconds, $\frac{T}{4} = 5.94$ and $\delta = 0.06$. Hence

$$A = \sqrt{2}\,\frac{L}{G}(1.008).$$

If $\delta = 0$, $A = \sqrt{2}\,\frac{L}{G}$, consequently an error of 1 per cent. in T causes an error of 0.8 per cent. in A.

To make sure that the observed radiation pressures depended only on the intensity of the beam, and were uninfluenced by the wave-length of the incident energy, the ballistic observations of pressure, the thermal measurements of intensity, and the determination of the reflection coefficients, were carried out for three entirely different wave-groups of the incident radiation. In the measurements designated "through air," no absorbing medium was introduced in the path of the beam between the lamp and the balance except the glass lenses and plates already mentioned. In the measurements "through red glass," a plate of ruby glass was put in the path of the beam between L_2 and d_3 (Fig. 3). For the observations

TABLE II.

August 28. *Light Pressure. Ballistic Measurements. Air.*

Surface.	Zero.	Throw.	Deflection.	Lamp.	Deflection (Lamp 100).	
					E.V.	*W.V.*
			mm.	mm.	mm.	mm.
EVC_s	281.4	248.5	32.9	164.3	20.0	
WVD_s	281.5	313.9	32.4	164.5		19.7
EVC_s	281.4	249.8	31.6	157.9	20.0	
WVD_s	281.5	310.5	29.0	147.0		19.8
EVC_s	281.5	252.6	28.9	144.8	20.0	
WVD_s	281.5	309.6	28.1	141.8		19.8
EVC_s	281.5	252.9	28.6	143.5	19.9	
WVD_s	281.5	309.3	27.8	140.4		19.8
				Average	19.97	19.77
				Average, $\frac{C_s+D_s}{2}$	19.87	
			Magnet Reversed.			
EVD_g	280.1	246.0	34.1	180.4	18.92	
WVC_g	280.0	317.8	37.8	187.3		20.20
EVD_g	279.8	247.2	32.6	170.8	19.20	
WVC_g	279.4	313.7	34.3	169.4		20.25
EVD_g	279.1	248.9	30.2	161.1	18.80	
WVC_g	279.0	311.9	32.9	161.6		20.35
EVD_g	279.0	249.1	30.0	158.9	18.90	
WVC_g	278.9	311.2	33.2	164.4		20.20
				Average	18.97	20.25
				Average, $\frac{C_g+D_g}{2}$	19.61	

TABLE III.

Radiation Pressure. Ballistic Measurements, through Air.

Date.	Air Pressure in mm. of Hg.	Balance Period T.	Sens. of Galvan'r. G_s (Silver).	Sens. of Galvan'r. G_g (Glass).	l = Lever Arm. cm.	$\frac{C_s + D_s}{2} = P_s$.	$\frac{C_g + D_g}{2} = P_g$.	P_s Corrected for $T = 24''$.	P_g Corrected for $T = 24''$.	$\frac{P_s \times G_s}{l}$.	$\frac{P_g \times G_g}{l}$.	Average.
June 19	32.5	23.75	[734]		.814	Average = 18.88		Average = 18.73		Average = 16.89		16.89
" 20	32.5	23.75	756	768	.814	19.67	16.94	19.51	16.81	18.12	15.86	17.00
" 23	37.0	23.75	700	716	.814							
July 23	16.0	23.75	682	707	.831	21.16	20.42	21.00	20.26	17.25	17.25	17.25
" 25	16.6	23.75	684	684	.815							
" 26	16.6	23.75	720	710	.815	19.34	19.98	19.18	19.82	16.93	17.28	17.10
Aug. 27	16.8	23.82	724	710	.823	20.16	19.40	20.00	19.25	17.60	16.61	17.10
" 28	15.7	23.82	721	716	.824	19.87	19.61	19.73	19.48	17.26	16.97	17.10
" 29	13.7	23.82	712	701	.824	19.68	20.07	19.53	19.92	16.90	16.97	16.94
" 31	14.0	24.00	718	713	.810	18.55	18.94	18.55	18.94	16.44	16.60	16.52
Sept. 1	16.6	24.00	692	672	.808	19.14	20.17	19.14	20.17	16.40	16.78	16.59
" 20	16.4	23.78	670	676	.812	20.96	20.02	20.81	19.87	17.17	16.54	16.86
" 23	16.4	23.78	666	684	.816	21.32	20.27	21.16	20.12	17.27	16.87	17.07
" 24	16.2	23.78	667	669	.816	20.76	19.80	20.60	19.65	16.84	16.11	16.47
									Average,	17.11	16.71	16.91 ± 0.053

"through water cell," a 9-mm. layer of distilled water in a glass cell was placed in the path of the beam at the same point.

The separate observations entering into a single series of ballistic measurements and their treatment will appear from Table II., which is copied direct from the laboratory note-book and represents an average ballistic series. The designations EVC_s, WVD_s, EVD_g, and

TABLE IV.

Radiation Pressure. Ballistic Measurements.

Date.	$\frac{C_s + D_s}{2}$.	$\frac{C_g + D_g}{2}$.	P_s Corrected for $T = 24''$.	P_g Corrected for $T = 24''$.	$\frac{P_s \times G_s}{l}$.	$\frac{P_g \times G_g}{l}$.	P Average.
Through Water Cell.							
June 20	18.62	17.10	18.46	16.96	17.14	16.00	16.57
July 25	19.00	20.10	18.85	19.94	15.82	16.74	16.28
" 26	18.03	19.39	17.89	19.33	15.80	16.84	16.32
Aug. 27	18.63	18.66	18.50	18.53	16.29	15.99	16.14
" 29	18.25	19.02	18.10	18.87	15.68	16.06	15.87
Sept. 20	20.39	19.14	20.23	19.00	16.69	15.82	16.25
" 23	20.21	19.51	20.05	19.36	16.37	16.23	16.30
" 24	19.84	18.91	19.69	18.77	16.10	15.40	15.70
				Average.....	16.24	16.15	16.20 ±0.066
Through Red Glass.							
June 23	19.99	18.40	19.83	18.26	17.05	16.06	16.56
July 25	20.70	20.94	20.54	20.77	17.24	17.43	17.33
Aug. 27	19.97	19.25	19.82	19.10	17.46	16.46	16.96
" 28	19.99	19.42	19.84	19.28	17.36	16.75	17.05
" 29	19.99	19.92	19.84	19.77	17.14	16.82	16.98
" 31	18.98	19.14	18.98	19.14	16.82	16.84	16.83
Sept. 20	21.00	19.97	20.83	19.82	17.19	16.50	16.84
" 23	21.48	20.34	21.31	20.18	17.39	16.92	17.15
" 24	21.00	19.68	20.83	19.53	17.03	16.03	16.53
				Average.....	17.18	16.65	16.91 ±0.051

WVC_g, mean that the vane C in the first case was on the east side of the rotation axis with its silver face toward the light. The subscript g signifies that the glass face of the vane was toward the light. The second column of the table gives the zero reading of the balance before opening the shutter; the third, the end of the swing produced by a six-second exposure; the fourth, the deflec-

tion of the balance; the fifth, the ballistic deflection of the lamp galvanometer G_2. Columns six and seven give the balance deflection reduced to standard lamp.

The results of all the ballistic pressure measurements "through air," are collected in Table III. In the fourth and fifth columns two values are given for the constant of the lamp galvanometer G_2, since reversing the magnet on the balance bell-jar to reverse the suspension within affected the constant of the galvanometer slightly. The values for the silver and glass faces forward were never the same. The subscripts show to which series, silver or glass, the constant belongs. The values of the lever-arm l of the balance in the sixth column, are obtained by measuring the distance between the centers of the images when on the east and west vanes (by the dividing engine T_2, Fig. 3) and dividing by two. The columns headed $\frac{C_s + D_s}{2} = P_s$ and $\frac{C_g + D_g}{2} = P_g$ are the average moments due to pressures for the silver and glass sides of the vanes respectively toward the light. The next two columns contain these moments corrected for a period of twenty-four seconds of the torsion balance. The columns headed $\frac{P_s \times G_s}{l}$ and $\frac{P_g \times G_g}{l}$ are the corresponding forces reduced to standard sensitiveness, $G = 1{,}000$. The final column contains the averages of the two columns which precede it. Table IV. exhibits corresponding data for "red glass" and "water cell." The air pressure, period of the balance, lever arm and galvanometer constants are those given in Table III. for the same date.

In these ballistic measurements the lamp reading was the throw due to an exposure of the light upon the bolometer for six seconds, but in the energy measurements the lamp reading was a stationary deflection due to prolonged exposure. To bring the pressure values into comparison with the energy measurements it is necessary to reduce the average of the quantities in the last column to pressures in dynes by multiplying by 0.363×10^{-5}, the torsion coefficient of the quartz fiber, and to reduce not only to a static deflection of the torsion balance but also to a static deflection of the lamp galvanometer G_2. The ratio of a ballistic to a static deflection

of the galvanometer G_2 was obtained from a long series of lamp exposures. This ratio was found "through air" to be = 1.55; "through red glass" = 1.535; "through water cell" = 1.502. These differences are probably due not solely to the damping constant of the galvanometer but to the peculiar manner in which the bolometer was warmed up to its stationary conditions by the beam from the lamp. Applying these reduction factors to the averages in Tables III. and IV., we obtain the following results. The pressure of the standard light beam which has passed

(*a*) through air $= 16.91 \times \frac{1.55}{1.357} \times 0.363 \times 10^{-5}$

$= (7.01 \pm 0.023) \times 10^{-5}$ dynes;

(*b*) through red glass $= 16.91 \times \frac{1.535}{1.357} \times 0.363 \times 10^{-5}$

$= (6.94 \pm 0.024) \times 10^{-5}$ dynes;

(*c*) through water cell $= 16.20 \times \frac{1.502}{1.357} \times 0.363 \times 10^{-5}$

$= (6.52 \pm 0.028) \times 10^{-5}$ dynes.

THE ENERGY MEASUREMENTS.

Before rejecting the bolometer method used in the preliminary measurement of energy, a second bolometer of slightly different construction was tried; but the lack of uniformity of resistance, already mentioned, made its indications too uncertain for the present work. The radiant intensity of the beam used in the later experiments was determined by directing it upon the blackened face of a silver disc, weighing 4.80 grams, of 13.3 mm. diameter and of 3.58 mm. thickness, and by measuring its rate of temperature rise as it passed through the temperature of its surroundings. The disc was obtained from Messrs. Tiffany & Co. and was said by them to be 99.8 per cent. fine silver. Two holes were bored through parallel diameters of the disc, one fourth of the thickness of the disc from either face. Two iron-constantan thermo-junctions, made by soldering 0.1 mm. wires of the two metals, were drawn through the holes

into the center of the disc. To insulate the wires from the disc, fine drawn glass tubes were slipped over them and thrust into the holes, leaving less than 2 mm. bare wire on either side of the junctions. The wires were sealed into the tubes, and the tubes into the disc by solid shellac. The tubes projected 15 mm. or more from the disc and were bent upward in planes parallel to the faces of the disc.

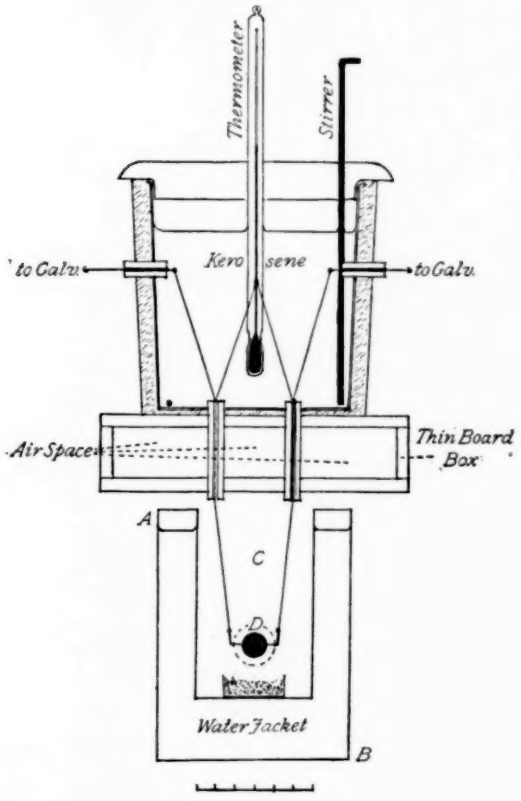

Fig. 6.

The general arrangement will be seen in Fig. 6. The disc was suspended by the four wires some distance below a small flat wooden box. On the box was fastened a calorimeter can swathed in cotton and filled with kerosene in which the constant thermo-junctions were immersed. Copper wires soldered to the two ends of the thermo-electric series were brought out of the colorimeter, and

the circuit was closed through 1,000 ohms. in series with the 500 ohms. resistance of galvanometer G_1. The thermo-junctions in the disc were in series, and as each junction was midway between the central plane of the disc and either face, it was assumed that when the disc was slowly warmed by heating one face the electromotive forces obtained corresponded to the mean temperature of the disc. One face of the disc was blackened by spraying it with powdered lampblack in alcohol containing a trace of shellac. This method was suggested by Professor G. E. Hale and gives very fine and uniform dead black coatings not inferior to good smoke deposits.

For the energy measurements the bell-jar and the torsion balance were removed from the platform P (Fig. 2) and a double-walled copper vessel, AB (Fig. 6), which served as a water jacket surrounding a small air chamber C, was mounted in the same place. A tube 2 cm. in diameter was soldered into the front face of the jacket to admit the light beam into the chamber C. This opening was covered by a piece of plate glass similar to the plates forming the larger windows in the bell-jar.

The needle system in G_1, a four-coil du Bois-Rubens galvanometer was suspended in a strong magnetic field so that its period was about four seconds. The system was heavily damped by a mica air-fan of large surface. The disc junctions and galvanometer responded quickly to the radiation, as was shown by the reversal of motion of the magnet system 1.2 seconds after the light was cut off from the disc when the latter was a few degrees above the temperature of the room.

(*Concluded in next number.*)

SOME OPTICAL PROPERTIES OF IODINE. III.

By Wm. W. Coblentz.

SINCE the publication of his previous work on iodine[1] the writer has been able to obtain very pure rock salt crystals from which cells could be made that were thin enough so that liquid substances could be explored up to 16 μ. At this point rock salt suddenly begins to absorb heavily[2] so that the exploration could not be extended beyond this point.

The cell walls were made by splitting the rock salt parallel to a cleavage plane. This gave thin plates that were quite plane, smooth, and of a finer polish than could be obtained by hand polishing. In fact the surface thus obtained is not so easily attacked by moisture and is of far more service in all work like the present. To make a cell, a copper wire, varying from .16 to .3 mm. in diameter, was bent into a U shape, and coated with Lapage's glue. This was pressed between two plates of rock salt and permitted to dry. It was found that the substances investigated did not attack the glue after it had dried. The size of the cell was usually about 1 × 2.5 cm. After it was filled, the top was covered with thick viscous glue. The cell was mounted on a heavy wooden block. This block moved in vertical ways, before the spectromoter slit, and had an opening about 4 × .5 cm. cut in it, through which the transmitted energy passed and over which the cell was securely mounted. A clear plate of rock salt was mounted directly below the cell, and all the remaining open spaces were covered with tinfoil. This arrangement prevented all radiation from getting into the radiometer except that which passed through the cell, or through the rock salt plate. A double-walled sheet-iron shutter was placed between the block and the intense source of energy, which was a

[1] Phys. Rev., XVI., Nos. 1 and 2, 1903.

[2] Rubens & Trowbridge, Wied. Ann., 60, p. 737, found a transmission of only 65 per cent. at 16 μ.

"heater" of a 110-volt Nernst lamp. In this work, however, a 90-volt storage battery system was employed, which gave a steady source of radiation.

The "heaters" consist of hollow cylinders of clay wound with platinum wire, the whole of which is covered with kaolin.

The extraordinary distribution of energy from these heaters is shown in Fig. 22. The first maximum is no doubt due to the hot

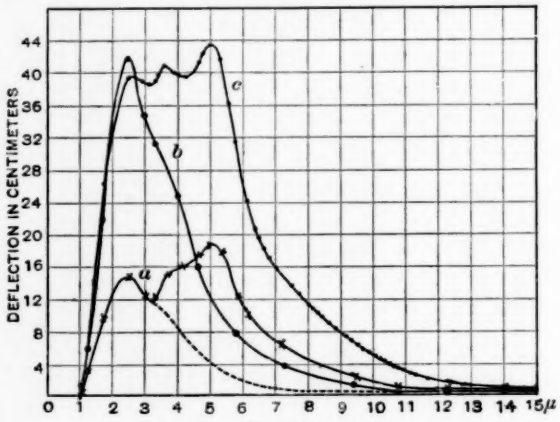

Fig. 22.

platinum wire, while the third maximum lies close to the absorption band of quartz found [1] at 5.3 μ. The minima lie close to the well-known atmospheric absorption bands of H_2O and CO_2 found [2] 2.7 and 4.4 μ.

The suppression of the selective radiation at 5.3 μ and 8.0 μ is well illustrated in curve *b*. The film of Fe_2O_3 was obtained by dipping the "heater" in a $FeSO_4$ solution, which was then oxidized by heating.[3] The dotted part of *a* indicates the probable radiation from the platinum wire which is wound on the heater. For curves *a* and *b* the spectrometer slit was .3 mm. wide. In actual work on absorption a wider slit (1 mm.) was used at the source, and a .4 mm. slit at the radiometer. Curve *c* represents the distribution of *a* for the latter case. With this slit-width the maximum of *b* is about 50 cm. deflection, but in the region beyond 8 μ the ordinates are not much higher than for the present curve. It will thus be seen that for work

[1] E. F. Nichols, PHYS. REV., 4, 1897.
[2] Paschen, Wied. Ann., 52, p. 223, 1894.
[3] Paschen, Wied. Ann., 56, p. 762, 1895.

like the present, where large deflections are necessary, from 7 to 16 μ, the unmodified heater is the better. Curve *c* was obtained while finding the absorption of a certain compound, and is of interest, since, by its regularity, it shows the constancy of the radiation.

The radiation from this heater was found when covered with borax, also when a strip of mica .3 mm. in thickness was wound on it, but in neither case was the energy curve different from the original. The mica did not show emission minima at 8.4 μ and 9.4 μ as computed by Rosenthal.[1] However, the strip may have been too thin to show this effect.

The method of making observations consisted in noting the radiometer deflection for the transmission through the cell, and then, on raising the block until the rock salt plate was before the slit, that deflection was read. The ratio of the deflection through the cell to that through the rock salt gave the transmission immediately, and more accurately than by finding the absorption of the empty cell. The transmission of the solvent was first found, then, without disturbing the position of the mounted cell, that of the solution of the substance under investigation. It was found that if the cell was changed in its mounting, while making a series of observations, errors were introduced in the work. From the transmission through the solvent and the solution two curves are obtained, from which one can obtain the absorption due to the substance dissolved.

The exploration of the iodine solution in CS_2 was undertaken to learn whether transparency continued in the region of 7.3 μ where solid iodine has a heavy absorption band. From the following curves it will be noticed that no such absorption occurs. In other words, the selective adsorption as well as general absorption, observed in solid iodine, disappears entirely when dissolved in carbon bisulphide or chloroform.

In the first part of this work mention was made of the well-known fact that iodine is brown in color when dissolved in alcohols, acids, ethers and aqueous solutions of metallic salts; that it imparts a red tint to benzine, toluene, etc.; and that in chloroform, carbon bisulphide, etc., the solutions are violet. Most of the above-named compounds have been investigated by Dutoit,[2] using a quartz prism.

[1] Rosenthal, Wied. Ann., 68, p. 782, 1899.

[2] Dutoit, Bull. Soc. Vaudovise des Sci. Naturelle, 38, pp. 1–28, 1902.

His explorations extent to 2.5 μ. He found a great similarity for the absorption of the brown solutions and also among those that are violet. He and the writer disagree however in that the former finds that the transparency of the brown and the violet solutions begins at about 1.8 μ, while the latter finds that the violet solutions become transparent at less than 1.4 μ and that the brown solutions continue to absorb beyond 2.5 μ. This is of minor importance, however, since Dutoit used sunlight for his source of energy, which was very unsteady, and may have introduced slight errors. Then the purity of the materials must also be considered. For ordinary ethyl alcohol containing water the writer found transparency to begin at 1.25 μ (Fig. 10).

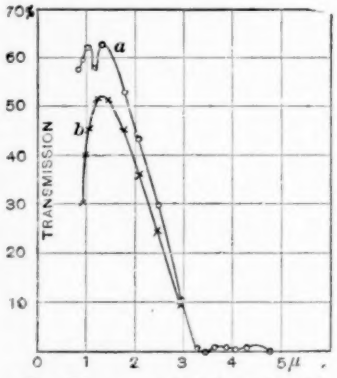

Fig. 23. a = glacial acetic acid; b = sat. sol. of iodine in same acid.

For the present work absolut eethyl alcohol was selected to represent the brown, acetic acid the reddish-brown, and chloroform and carbon bisulphide the violet solutions.

In Fig. 23 is given the transmission curve of a layer of glacial acetic acid .18 mm. thick, and also the curve for a saturated solution of iodine in this compound. The iodine solution absorbs heavily throughout the whole region. Both become opaque at 4 μ, probably due to the presence of water.

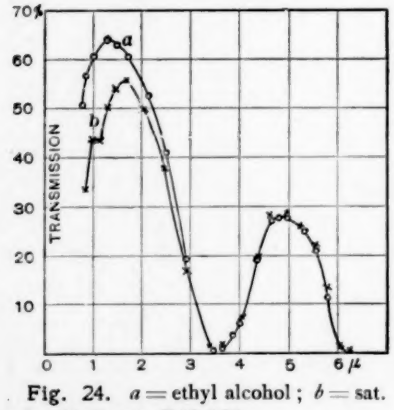

Fig. 24. a = ethyl alcohol; b = sat. sol. of iodine in C_2H_5OH.

With the present source of energy it is as difficult to work at 1 μ as at 12 μ, so that no at attempt was made to locate accurately the absoption band at 1.1 μ. At this point a prism having greater dispersion is more useful.

In Fig. 24 is given the curve for a saturated solution of iodine in absolute ethyl alcohol. Here the absorption is also heavy and

extends to 3 μ. Beyond this point the solution becomes transparent. Table I. gives the transmission for these two solutions.

TABLE I.

Transmission through a saturated solution of iodine in absolute ethyl alcohol and in glacial acetic acid.

Wave-lengths in μ.	Alcohol, C_2H_5OH	Sat. Sol. of Iodine in C_2H_5OH	Acetic Acid, $C_2H_4O_2$	Sat. Sol. of Iodine in $C_2H_4O_2$	Wave-lengths in μ.	C_2H_5OH	Cell .18 mm. Thick.		
							C_2H_5OH + Iodine.	$C_2H_4O_2$	$C_2H_4O_2$ + Iodine.
.92	51		58	30	3.68	1.0	1.35	.34	0
.984	56.5	33	59	40	3.92	3.51		.2	0
1.06	61	44	62.3	45	4.05	6.97	6.8	0	0
1.16	68	43	57.5	52	4.35	18.8	19.2	.41	0
1.28	64	50	63	45	4.65	27.2	28.3		
1.46	63.2	54			4.8	27.6			
1.72	60.3	56.3	53	45.3	4.95	27.5	29.0	.42	0
2.03	52.5	50	43.4	36	5.28	25.1	26.1		
2.50	41	38	30	24.7	5.53	21.2	22.0	0	0
2.93	19.5	16.5	10.5	9.8	5.80	11.5	13.8		
3.30	1.4	1.2	.2	0	6.03	.34	.61		
3.49	.34		0	0	6.27	.25	.34		
					6.78	00	00		

The violet solutions present a very different appearance. The curves for CS_2, and for a saturated solution of iodine in CS_2, in Fig. 25 coincide throughout the whole region from 1.5 to 16 μ. The

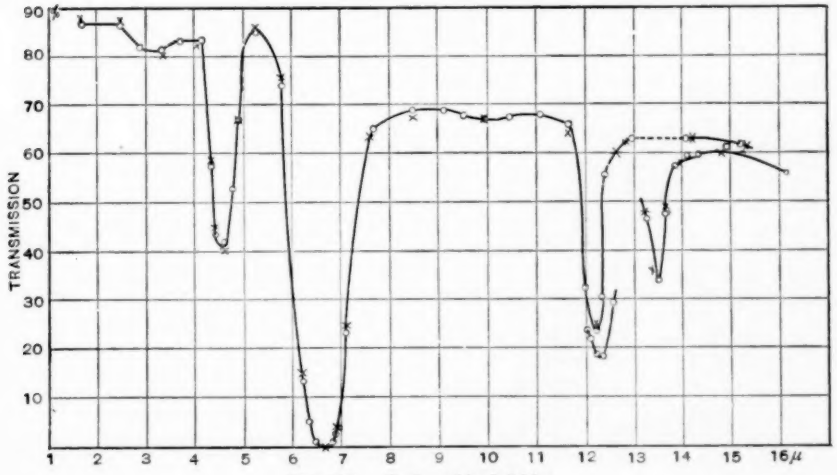

Fig. 25. Carbon bisulphide.

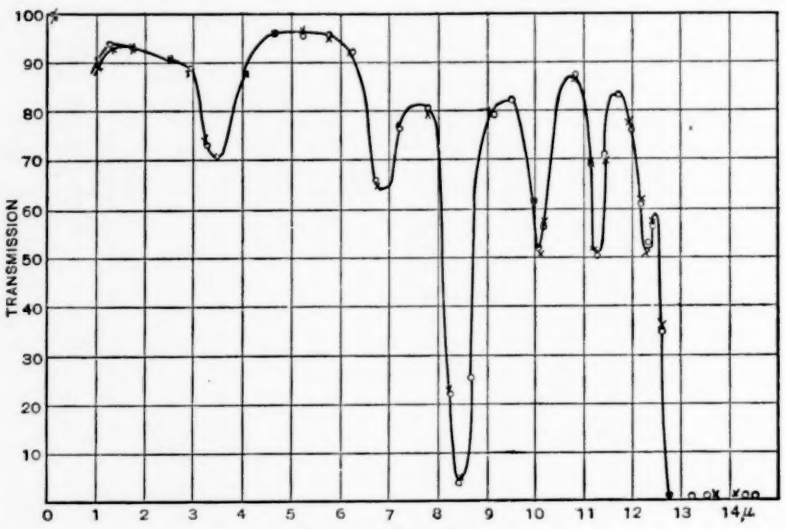

Fig. 26. Chloroform.

TABLE II.

Transmission through a saturated solution of iodine in CS_2.

Wave-lengths in μ.	CS_2	CS_2 + Iodine.	Wave-lengths in μ.	CS_2	Cell 3 mm. Thick. CS_2 + Iodine.
1.72	86.5	87.0	8.68	69.4	
2.50	86.8	87.6	9.12	69.2	
3.30	80.5	80	9.50	68.0	
3.68	83		9.88	67.5	67.4
4.05	83.4	83.7	10.50	68	
4.35	57.8	59.0	11.14	68	
4.48	43.3	46.2	11.17	66.7	64.4
4.65	41.6	40.8	11.95	32.6	
4.80	53.6		12.18	23.8	26.0
4.95	67.8	67.3	12.3	30.8	
5.28	85.0	85.6	12.4	57.0	
5.80	74.2	76.0	12.6	61	
6.27	14.0	14.3	13.0	63.2	62.5
6.40	5.4		14.1	62.7	62.8
6.50	1.15		Cell .18 mm. thick.		
6.60	.54	.48	13.32	47	48
6.78	.86		13.55	34	36
6.87	1.0		13.75	48	49
7.00	3.64	4.7	13.9	57.5	58
7.26	24	25.4	14.45	59	
7.80	65.3	66.3	14.8	59	60
8.25	66.4		15.3	59	
			16.1	56	

same coincidence occurs for the chloroform solution (Fig. 26), which has been indicated as absorbing a little at 1 μ. Here transparency begins at less than 1.4 μ. Tables II. and III. give the transmission for these two solutions.

TABLE III.

Transmission through a saturated solution of iodine in chloroform.

Wave-lengths in μ.	Chloroform $CHCl_3$	$CHCl_3$ + Iodine.	Wave-lengths in μ.	$CHCl_3$	Cell .16 mm. $CHCl_3$ + Iodine.
1.06	90.0	89.0	9.50	82.4	
1.28	93.4	92.8	9.88	61	61.2
1.72	92.3	92.2	10.02	51.7	51
2.50	90.3	90.0	10.18	56.4	57.3
2.93	88.4	87.6	10.5	82.0	82.5
3.3	73	73.3	10.8	87.4	85.6
4.05	87.2	87.5	11.14	69.0	69.0
4.65	95.4	95.3	11.28	50	51
5.28	95.0	95.8	11.44	70.8	70
5.80	95.0	94.3	11.70	83.5	
6.27	81.4	81.0	11.95	77	79
6.78	65.4	64.9	12.18	61	61.5
7.26	76.8	77.2	12.30	53	50.5
7.8	80.2	79.8	12.4	56.5	58
8.25	22.4	22.8	12.6	35.0	36
8.48	4.4	4.03	12.8	00	00
8.68	25.5		13.0	00	00
9.12	79.5	80.5	14.0	00	00

The carbon bisulphide curves show absorption bands with transmission minima at 3.1, 4.65, 6.7, 12.25 and 13.5 μ. The chloroform curves show minima at 3.35, 6.8, 8.5, 10.05, 11.35 and 12.26 μ.

SUMMARY.

The cause of the variation of the color of iodine in different compounds and solutions has been the subject of numerous investigations. Krüss and Thiele,[1] also Bechman,[2] have found that iodine solidified in various solvents, *e. g.*, ether, chloroform and benzine has a molecular weight = I_2 for dilute solutions. Variations were irregular both for brown and for violet solutions. Concentrated solu-

[1] Krüss and Thiele, Zeit. Anorg. Chemie, 7, 25 (1894). Zeit. Phys. Chemie, Vol. VIII., p. 52.

[2] Bechman, Zeit. Phys. Chemie, Vol. V., p. 76, 1889.

tions usually gave high molecular weights. Gauthier and Charpy[1] found that brown solutions $= I_4$ but gradually break up to I_2 in becoming violet solutions, which is equivalent to iodine vapor. Paterno and Nasini[2] found I_2 for the molecular weight of iodine in acetic acid, for dilute solutions only, and a more complex structure in concentrated solutions. Finally Loeb[3] working on the vapor tensions of iodine solutions found I_4 in brown solutions, and I_2 to I_3 in violet solutions.

The most recent work on this subject is that of Lachman[4] who finds that iodine solutions have but two colors, red and brown, when made with pure solvents. "Saturated solvents give violet; solvents which have unsaturated character give brown solutions." The latter are oxygen compounds. Some of the brown solutions are known to contain periodides. Baeyer and Villiger[5] have recently demonstrated the additive powers of combined oxygen atoms in oxygen compounds. From these facts Lachman assumes that all brown iodine solutions contain some addition product (molecule-solvent + I_2), while violet solutions contain simple iodine molecules. Since brown solutions tend to become violet when heated, and conversely, violet solutions become brown on being cooled; also, since heat produces dissociation, and cold induces association, the facts are in fair agreement with expectation.

Martens[6] has studied the influence as the absorption of C, S, Cl, Br and I in the ultra-violet. He found that the wave-length corresponding to the principal absorption band in the ultra-violet is approximately proportional to the square root of the atomic weight.

The present work deals with the absorption of iodine solution from the ultra-violet to 16 μ in the infra-red.[7] The brown solutions

[1] Gauthier and Charpy, Compt. Rend , Vol. I., p. 189, 1890.

[2] Paterno and Nasini, Berliner Ber., Vol. II., p. 2153, 1888.

[3] Loeb, Zeit. Phys. Chem., Vol. VII., p. 606.

[4] Lachman, A Probable Cause of the Different Colors of Iodine Solutions, Jour. Amer. Chem. Soc., Vol. XXV., No. 1, p. 50, 1903.

[5] Baeyer and Villiger, Ber. d. Chem. Ges., 34, 2679 (1901).

[6] Martens, Deutsch. Phys. Gesell., Verh. 4, p. 138, 1902.

[7] The refractive indices for wave-lengths from 9 to 12 μ, used in this work, were the values computed by Rubens (Wied. Ann., 53, 1894). They were used instead of the observed values given by Rubens and Trowbridge (Wied. Ann., Vol. 60), because they gave a smooth curve with those at 8 μ, 20.57 μ and 22.3 μ while the observed values

were found to be quite transparent at the red end of the spectrum, and to absorb heavily in the violet. The violet solutions absorb heavily in the visible, and transmit violet and all the infra-red beyond 1.2 μ.

A study of the absorption spectra of iodine in solution does not tell us the exact nature of the structure of iodine in these solutions, but from the work just quoted it must be conceded that this very different behavior of the brown and violet solutions is due, in part at least, to the variation in the molecular structure of iodine when dissolved in these two classes of solvents. Few substances have been so thoroughly investigated, yet from a chemical as well as a physical standpoint, the question of the exact composition of iodine in solution remains unsolved.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,

February, 1903.

did not. Since then it has been found that these observed indices in Vol. 60 are wrong or misprints, and that the correct values are given in an inconspicuous, unindexed notice in Wied. Ann., 61, p. 224, 1897, which has escaped the attention of at least one person besides the writer. The whole is unfortunate. As a result of this the wave-lengths are too large by .29 μ at 9 μ, by .4 μ at 10.57 μ and by .16 μ at 13 μ, beyond which they remain almost as plotted. Furthermore the correction for lack of minimum deviation is nowhere greater than .08 μ instead of the largest value given in the table. A final correction will be given in the future. April, 1903.

NEW BOOKS.

A Manual of Physical Measurements. By JOHN O. REED and KARL E. GUTHE. Ann Arbor, Michigan, George Wahr, publisher, 1902. Pp. 185.

This manual of laboratory physics is one of the class designed primarily to satisfy the requirements of the laboratory where it was prepared. It describes the work required of students in physics and electrical engineering in their first course in the physical laboratory at the University of Michigan. It is a laboratory manual in the proper sense, being planned for the student's use in work supplementary to a course of lectures and recitations in theoretical physics.

The book contains brief general instructions upon the objects and methods of laboratory work and computation ; it describes sixty-six exercises in one hundred and seventy-five pages, and there are ten tables of constants besides tables of four-place trigonometric functions and logarithms. The problems are distributed as follows : Twenty in mechanics, two in sound, five in heat, ten in light and twenty-eight in electricity. The criticism might be made upon this selection that there are too few exercises in mechanics, sound and heat.

The chapter on elasticity contains seven of the twenty problems in mechanics, and fills nearly half of the space allotted to this section. The treatment is excellent, but its fullness makes more apparent, by contrast, the lack of completeness in the other parts.

The optical measurements include the curvature and focal lengths of lenses, the magnifying power of a telescope, index of refraction with the microscope and the spectrometer, and wave-length with a transmission grating. The five exercises in heat are upon the fixed points of a thermometer, the water equivalent of a calorimeter, specific heat of a solid, and the heat of fusion and heat of vaporization of water. In sound there are only exercises on the velocity of sound by Kundt's method, and a graphic method for rating a tuning-fork. The twenty-eight exercises in electricity cover in an acceptable manner the usual laboratory exercises of this class, making this section the most satisfactory, because of its completeness.

The instructions give such theoretical discussions as experience shows are necessary to refresh the memory of the student. The practical opera-

tions are described methodically, not simply enumerated. A number of general descriptions are given upon important subjects such as length, time and mass, and electrical units and standards. Often the instructions seem to lack thoroughness, but since the authors state "the book being designed for beginners, makes no claim to completeness either in subject matter or in exposition," this cannot be held as a valid criticism.

Of positive errors not many have been noticed, but there is one that must be referred to at length. The correct formula is given for computing the probable error of a series of observations, then the following explanation is made. "If e be the probable error of a set of readings, a the average, and x the true value of the reading sought, then

$$a + e > x > a - e;$$

which means that the true value of x lies between the mean *plus* the probable error and the mean *minus* the probable error." This erroneous idea of probable error is sometimes met with, but to find it so explicitly stated in a text-book is indeed serious. It is necessary to call the student's attention to the fact that the probable error is not a limiting value such that there is no probability of a greater error; and that it is a quantity such that in a large series there are certain to be just as many errors greater than it, as there are smaller ones. The real magnitude of the error such that there is only one chance in a thousand of a greater error, is nearly five times as large as the probable error.

A graphical method is given for correcting calorimeter observations for radiation, which is both incorrect and inconsistent with itself. Under thermometry it is stated that the fixed points of a thermometer should be frequently determined. In good practice the boiling point will never be determined more than once, and this information should be given to the student. The instructions are first to determine the ice point, and then the steam point; after this the ice point is to be redetermined. But the description does not mention any use to be made of the redetermined ice-point, while this is really the only determination that should be used in finding the fundamental interval.

There are several statements to which one might take exception; such, for instance, as that aluminum is the best material for the cup of a calorimeter, and that the most accurate method for measuring resistances is by means of the Wheatstone bridge, and that the proper method for focusing the telescope of a spectrometer is to remove it from its carrier and sight it on the moons of Jupiter or the planet Venus.

There appears to be no use made of the table of atomic weights, while its room might well be taken for a table of temperature corrections to barometric readings.

Forms of record are given for every exercise. These are not always systematic, and sometimes degenerate into a list of quantities to be observed, and they even contain a list of logarithms required. The reviewer believes that such forms are detrimental.

The typography is good except in one particular; letters used as symbols are given in ordinary roman characters instead of the universal italic form. This makes reading somewhat difficult. Many of the cuts are not as good as they should be, and the same may be said of the printing and stitching.

Perhaps it is not a reviewer's province to say what a book should be but rather to learn whether it is what its author intended it to be. As compared with other books of the class to which it avowedly belongs, this one, except as noted above, is unusually satisfactory. The selection and arrangement of exercises covers the subject in a well-balanced, though brief, manner, forming a coherent and logical course, such as has been proved to be generally acceptable. No innovations in method or apparatus are exploited. While greater precision of detail might seem desirable to many, yet for a short course (this one is stated to be for forty exercises of two hours each) given to general college classes, and for one in which elaborate apparatus is not used, this book is one of the most satisfactory manuals that has been prepared.

DAYTON C. MILLER.

The Physical Papers of Henry Augustus Rowland. Baltimore, The Johns Hopkins Press, 1902. Pp. xii + 704.

The papers of the late Professor Rowland have been collected into a volume by a committee of the faculty of the Johns Hopkins University under the editorial supervision of Professor Ames. The volume opens appropriately with the sympathetic and appreciative address by President T. C. Mendenhall which was delivered in commemoration of Rowland's services to science, at Baltimore in 1901 and which forms a fitting introduction to the scientific papers. The book closes with a description by Professor Ames of the Rowland dividing engines with an admirable set of drawings showing in detail the mechanism of these remarkable machines.

Rowland's scientific work is too well known to demand any detailed comment here. Some readers will doubtless regret the decision of the committee of editors to omit the tables of standard wave-lengths to the preparation of which he devoted ten years of his life. They would have added somewhat to the bulk of the already large volume but they would at the same time have added greatly to its usefulness. As it stands however this collection of physical papers is an imposing monument to the genius of the author and it will find its proper place upon our shelves, a

worthy companion piece to the collected memoirs of Maxwell, Tait, Rayleigh and Kelvin.

E. L. N.

Verhandlungen der ersten internationalen seismologischen Konferenz zu Strassburg. Ergänzungsband I., herausgegeben von G. GERLAND. Leipzig, Engelmann, 1902. Pp. viii + 439.

The first supplementary volume of the international seismological conference at Strassburg is devoted to geophysics. It contains in addition to descriptions of the systematic study of earthquakes, a number of papers of more direct interest to the physicist.

Professor Laska contributes a study of those peculiar movements of the horizontal pendulum which, in contradistinction to the temporary pulsations ascribable to the usual earthquake disturbances, are of long-continued duration; lasting in some instances without interruption for months at a time. Experience shows that certain pendulums are particularly subject to this class of disturbance while others at the same station remain in comparative quiescence. The author ascribes this peculiar behavior to the fact that various layers of the earth's crust transmit seismic waves of some given length freely and suppress others. When a pendulum happens to be in tune with the layer upon which it is placed, it in turn responds to the waves transmitted by this layer while other equally sensitive pendulums remain at rest. Such pendulums show little additional disturbance during times of earthquake while those commonly at rest, may respond to the earthquake waves to a marked degree.

Laska finds an intimate connection between barometric pressure and these disturbances. He considers them on the other hand to be uninfluenced by local winds and he calls attention to an as yet unexplained coincidence between magnetic storms which disturb the compass needle and movements of the horizontal pendulum.

A paper by Professor Straubel of Jena, deals with the means of illuminating chronograph sheets for the photographic registration of deflections. He favors the combination of a concave mirror behind a linear source of light, a diaphragm in front of the same and a cylindrical lens by means of which an image of the diaphragm is thrown upon the sensitized surface.

The specific intensities of the sources of light commonly available for such purposes were found to be as follows:

Petroleum lamp, .036; acetylene flame, .07; Welsbach mantle, .10; glow lamp, .80; Nernst lamp, 3.0.

An investigation of the advantages of a quartz-fluorite system as compared with optical glass, undertaken by Dr. Lehmann at the request of the author, showed that the actinic action upon the photographic surface

for those portions of the spectrum in which the glass exerts strong absorption, is so slight when any of the sources of light mentioned above are used, that nothing is to be gained by the substitution of quartz and fluorite for glass.

E. L. N.

Galvanic Batteries: Their Theory, Construction and Use. By S. R. BOTTONE. Whittaker & Co., London and New York, 1902. Pp. 376.

This book contains ten chapters. The first gives a brief account of the work of Sulzer, Galvani and Volta. The next four describe, in an elementary manner, the principles involved in the operation of a galvanic battery and explain the meaning of the more common electrical and mechanical units. They also treat of the various methods of connecting cells and differentiate between primary and secondary batteries. The sixth chapter covers 230 pages or over two thirds of the book. Its subject is "Tabulation of the Different Cells." It constitutes the most important part of the book and the author's idea in preparing it is evidenced in the preface where he says: "A description of every known battery of any practical use is given, along with data as to E.M.F., internal resistance and adaptability to particular requirements." There certainly is a large number described. Some of them have demonstrated their usefulness by surviving their inception. Commercial requirements, however, have standardized primary battery practice to such an extent that but few types are necessary to meet the various demands. This part of the book is interesting to read and will be of value to those who are developing primary cells. The list is not, however, complete. Two important omissions are descriptions of the Reichsanstalt pattern of standard Clark cell and of the Carhart low temperature-coefficient standard cell. The rest of the book is devoted to dry batteries, choice of batteries and details and construction of the more useful cells and storage batteries. The book has 144 illustrations and a complete index.

SAMUEL SHELDON.